scalar perturbations from inflationary magnetogenesis

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“cosmological magnetic fields” workshop, Bernoulli Center

May 10, 2024

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outline

- introduction
  - case for primordial magnetic fields
- the general model
  - observables calculated: spectrum and bispectrum
- no backreaction, axion inflation
  - comparison with previous literature
- conclusion

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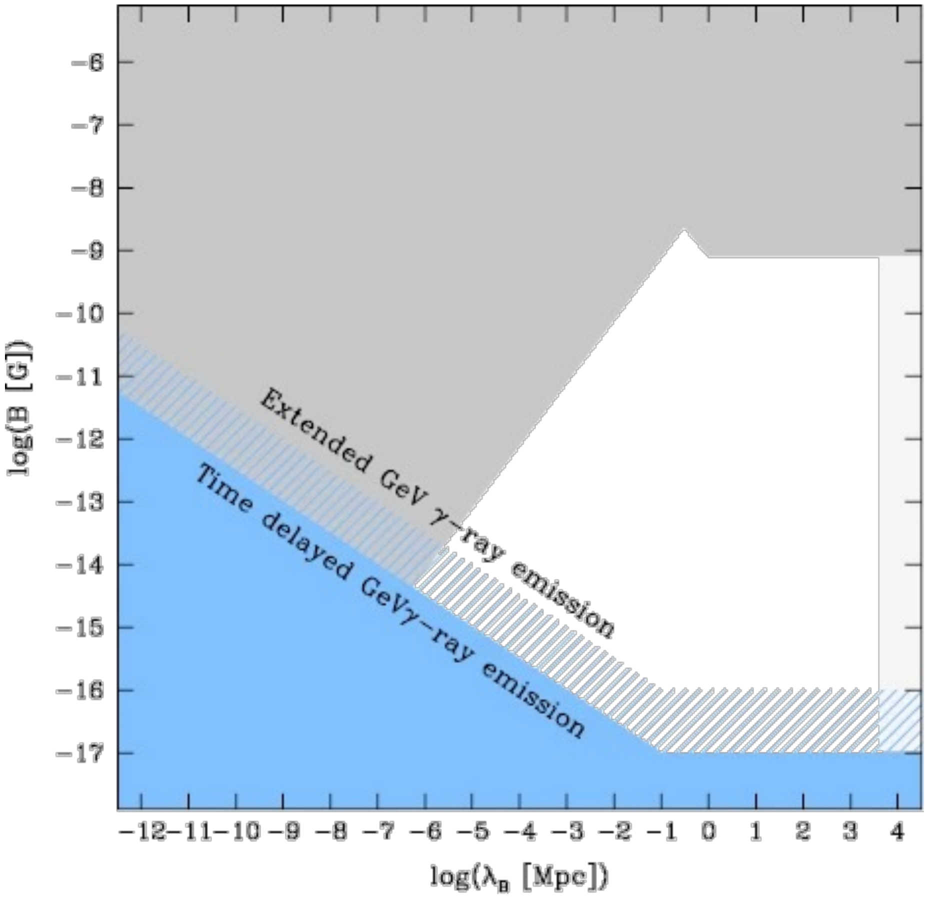
why consider magnetic fields?

- blazar spectra observations posit fields ~ 10^{-16}G at the scale of ~ 1 Mpc, with ~70% volume filling factor
- galactic fields ~ 10^{-6}G observed at z > 1, require seed fields for dynamo
- they could possibly have other cosmological effects, like on, for example, GW spectra, sound horizon, and scalar perturbations.



Durrer & Neronov 2013

where would they come from?

- early Universe
  - inflationary: nonconformal coupling of the EM fields with the inflaton
  - phase transitions: charge separation and turbulence during first order phase transitions

- late Universe: various instabilities, batteries and other plasma effects

could they be astrophysical?

- galactic fields could possibly have astrophysical origins
  - small scale dynamos could be important, requiring much weaker seed fields
  - plasma battery effects could have generated such weak seed fields
- blazar spectra could have astrophysical explanations
  - plasma instabilities could play a role
  - galactic winds, although reaching 70% filling factor could be difficult
- primordial fields could help with both!

more observations needed!

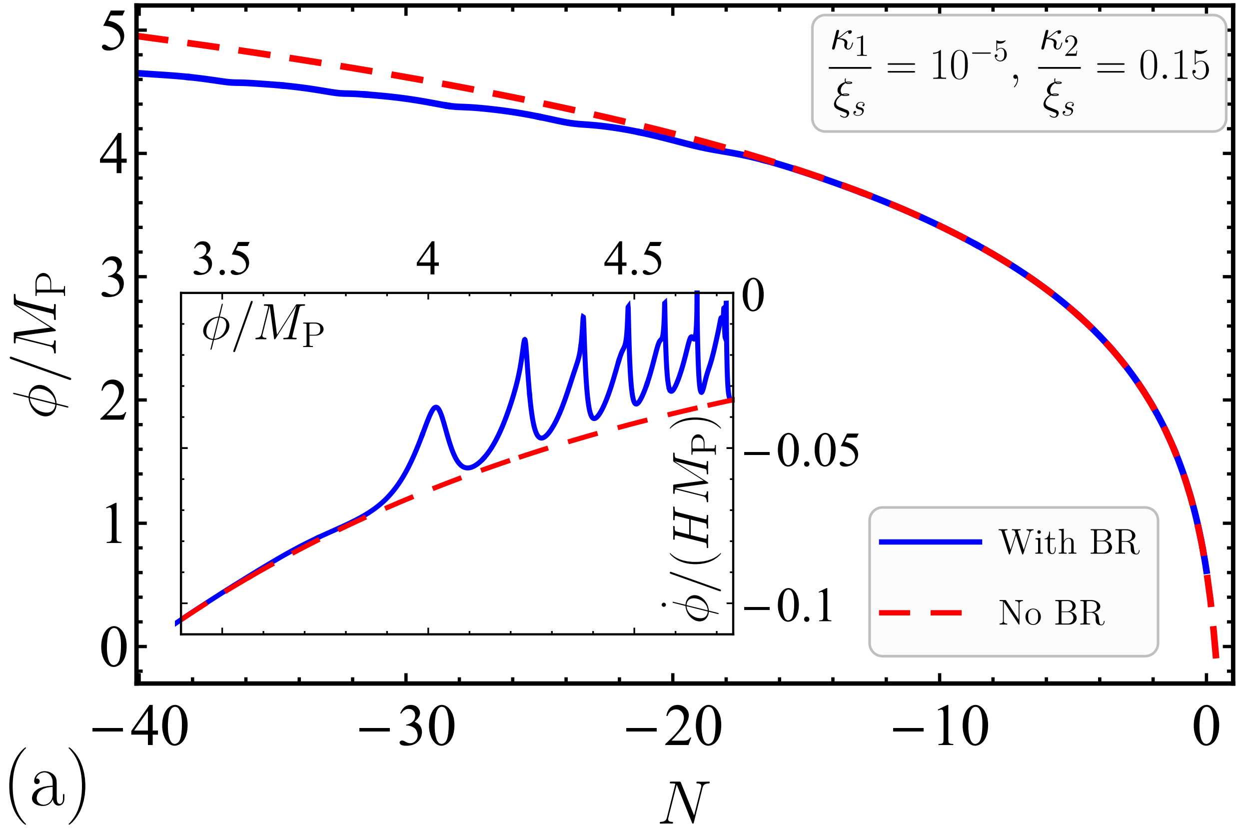
inflationary models

$$S = \int d^4x\sqrt{-g}\left[R + \frac{1}{2}(\partial\phi)^2 - V(\phi) + \frac{1}{4}I_1(\phi)F^2 + \frac{1}{4}I_2(\phi)F\cdot\tilde{F}\right]$$

- nonconformal coupling
- breaks spatial parity P explicitly in the general case

backreaction could be important

- even with low energy density, backreaction of the gauge field could be important for the evolution of the background inflaton field
- this necessitates a more self-consistent approach to understand the impact of gauge field on primordial perturbations



Durrer, Sobol & Vilchinskii 2023

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background evolution

$$\frac{a'^{\,2}}{a^2} \equiv \mathcal{H}^2 = \frac{1}{3M_{\mathrm{P}}^2}\left\{\frac{\phi_c'^2}{2} + a^2 V(\phi_c) + \frac{a^2}{2} I_1(\phi_c)[\langle \boldsymbol{E}^2\rangle + \langle \boldsymbol{B}^2\rangle]\right\}$$

$$2\mathcal{H} + \mathcal{H}^2 = -\frac{1}{M_{\mathrm{P}}^2}\left\{\frac{\phi_c'^2}{2} - a^2 V(\phi_c) + \frac{a^2}{6} I_1(\phi_c)[\langle \boldsymbol{E}^2\rangle + \langle \boldsymbol{B}^2\rangle]\right\}$$

$$\phi_c'' + 2\mathcal{H}\phi_c' + a^2 V'(\phi_c) = \frac{a^2}{2} I_1'(\phi_c)[\langle \boldsymbol{E}^2\rangle - \langle \boldsymbol{B}^2\rangle] + a^2 I_2'(\phi_c)\langle(\boldsymbol{E}\cdot\boldsymbol{B})\rangle$$

background evolution

- Maxwell's equations can be used to obtain

$$\langle \boldsymbol{E}^2\rangle' = -\left[4\mathcal{H} + 2\frac{I_1'(\phi_c)}{I_1(\phi_c)}\phi_c'\right]\langle \boldsymbol{E}^2\rangle - 2\frac{I_2'(\phi_c)}{I_1(\phi_c)}\phi_c'\langle(\boldsymbol{E}\cdot\boldsymbol{B})\rangle + 2\langle(\boldsymbol{E}\cdot \mathrm{rot}\,\boldsymbol{B})\rangle\,,$$

$$\langle \boldsymbol{B}^2\rangle' = -4\mathcal{H}\langle \boldsymbol{B}^2\rangle - 2\langle(\boldsymbol{E}\cdot \mathrm{rot}\,\boldsymbol{B})\rangle\,,$$

$$\langle(\boldsymbol{E}\cdot\boldsymbol{B})\rangle' = -\left[4\mathcal{H} + \frac{I_1'(\phi_c)}{I_1(\phi_c)}\phi_c'\right]\langle(\boldsymbol{E}\cdot\boldsymbol{B})\rangle - \frac{I_2'(\phi_c)}{I_1(\phi_c)}\phi_c'\langle \boldsymbol{B}^2\rangle - \langle \boldsymbol{E}\cdot \mathrm{rot}\,\boldsymbol{E}\rangle + \langle \boldsymbol{B}\cdot \mathrm{rot}\,\boldsymbol{B}\rangle\,.$$

gradient expansion formalism!

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perturbation evolution

- assumptions
  - longitudinal gauge for the metric
  - Coulomb gauge for the gauge field
  - vector and tensor perturbations neglected
  - vanishing slow roll parameters
  - quantum initial conditions

- small parameters
  - Bardeen potentials, Φ, Ψ
  - perturbations, $\delta\varphi$, $\delta_{P_i Q_j}$ $\forall\, \boldsymbol{P}, \boldsymbol{Q} \,\epsilon \{\boldsymbol{E}, \boldsymbol{B}, rot\ \boldsymbol{E}, rot\ \boldsymbol{B}, rot^2 \boldsymbol{E}\}$

EM perturbation source terms

- EM perturbations arise in specific combinations

$$F_\rho = \frac{a^2}{2M_P^2}\delta T_0{}^{0\,(\mathrm{EM})} = \frac{a^2 I_1(\phi_c)}{4M_P^2}\left[\delta_{\boldsymbol{E}^2} + \delta_{\boldsymbol{B}^2}\right],$$

$$F_v = \frac{a^2}{2M_P^2}\frac{\partial_i}{\triangle}\delta T_i{}^{0\,(\mathrm{EM})} = -\frac{a^2 I_1(\phi_c)}{2M_P^2}\frac{\partial_i}{\triangle}\varepsilon_{ijk}\delta_{E_j B_k},$$

$$F_p = -\frac{a^2}{2M_P^2}\frac{1}{3}\delta^i_j\,\delta T_i{}^{j\,(\mathrm{EM})} = \frac{a^2 I_1(\phi_c)}{12M_P^2}\left[\delta_{\boldsymbol{E}^2} + \delta_{\boldsymbol{B}^2}\right] = \frac{1}{3}F_\rho,$$

$$F_\pi = -\frac{a^2}{2M_P^2}\frac{3}{2\triangle^2}\left(\partial_i\partial_j - \frac{1}{3}\delta^i_j\triangle\right)\delta T_i{}^{j\,(\mathrm{EM})}$$

$$= -\frac{3a^2 I_1(\phi_c)}{4M_P^2}\frac{\partial_i\partial_j}{\triangle^2}\left[\delta_{E_iE_j} + \delta_{B_iB_j}\right] + \frac{a^2 I_1(\phi_c)}{4M_P^2}\frac{1}{\triangle}\left[\delta_{\boldsymbol{E}^2} + \delta_{\boldsymbol{B}^2}\right].$$

R. Durrer, R. von Eckardstein, DG, K. Schmitz, O. Sobol, and S. Vilchinskii, arXiv:2404.19694

EM perturbation source terms

$$F'_\rho = \Big[-2\mathcal{H} + \frac{I'_1(\phi_c)}{I_1(\phi_c)}\phi'_c\Big]F_\rho + \triangle F_v - \frac{a^2\phi'_c}{2M_\mathrm{P}^2}[I'_1(\phi_c)\delta_{\boldsymbol{E}^2} + I'_2(\phi_c)\delta_{\boldsymbol{E}\cdot\boldsymbol{B}}]$$
$$- \frac{a^2 I_1(\phi_c)}{2M_\mathrm{P}^2}\Big\{\langle\boldsymbol{E}^2\rangle\Big[\frac{I'_1(\phi_c)}{I_1(\phi_c)}\delta\varphi' + \Big(\frac{I''_1(\phi_c)}{I_1(\phi_c)} - \frac{I'^2_1(\phi_c)}{I_1^2(\phi_c)}\Big)\phi'_c\delta\varphi - (\Phi+\Psi)'\Big]$$
$$+ \langle(\boldsymbol{E}\cdot\boldsymbol{B})\rangle\Big[\frac{I'_2(\phi_c)}{I_1(\phi_c)}\delta\varphi' + \Big(\frac{I''_2(\phi_c)}{I_1(\phi_c)} - \frac{I'_1(\phi_c)I'_2(\phi_c)}{I_1^2(\phi_c)}\Big)\phi'_c\delta\varphi + \frac{I'_2(\phi_c)}{I_1(\phi_c)}\phi'_c(\Phi+\Psi)\Big]$$
$$+ 2\langle(\boldsymbol{E}\cdot\mathrm{rot}\,\boldsymbol{B})\rangle(\Phi+\Psi)\Big\},$$

$$F'_v = -2\mathcal{H}F_v + \frac{2}{3}\triangle F_\pi + \frac{1}{3}F_\rho - \frac{a^2}{2M_\mathrm{P}^2}\Big\{\frac{1}{3}\langle\boldsymbol{E}^2\rangle\big[I'_1(\phi_c)\delta\varphi - I_1(\phi_c)(\Phi+\Psi)\big] + \langle(\boldsymbol{E}\cdot\boldsymbol{B})\rangle I'_2(\phi_c)\delta\varphi$$
$$- \frac{2}{3}\langle\boldsymbol{B}^2\rangle\big[I'_1(\phi_c)\delta\varphi + I_1(\phi_c)(\Phi+\Psi)\big]\Big\},$$

$$F'_\pi = \Big[-2\mathcal{H} + \frac{I'_1(\phi_c)}{I_1(\phi_c)}\phi'_c\Big]F_\pi$$
$$- \frac{a^2 I_1(\phi_c)}{2M_\mathrm{P}^2}\Big(\frac{\delta_{ij}}{\triangle} - 3\frac{\partial_i\partial_j}{\triangle^2}\Big)\Big[\frac{I'_1(\phi_c)}{I_1(\phi_c)}\phi'_c\delta_{E_iE_j} + \frac{I'_2(\phi_c)}{I_1(\phi_c)}\phi'_c\delta_{E_iB_j} + \delta_{(\mathrm{rot}\,\boldsymbol{E})_iB_j} - \delta_{(\mathrm{rot}\,\boldsymbol{B})_iE_j}\Big].$$

modified perturbation variable

- ζ variable gets modified by the EM perturbations

$$\zeta = \Phi + \frac{\mathcal{H}}{\mathcal{H}^2 - \mathcal{H}'}(\Phi' + \mathcal{H}\Psi)$$
$$= \Phi + \frac{\mathcal{H}}{\mathcal{H}^2 - \mathcal{H}'}\left(\frac{\phi_c'}{2M_\mathrm{P}^2}\delta\varphi + F_v\right),$$

$$\zeta'' + p\zeta' + q\zeta = r^{(v)}F_v + r^{(\rho)}F_\rho + r_1^{(\pi)}F_\pi + r_2^{(\pi)}F_\pi' + r^{(\boldsymbol{E}^2)}\delta_{\boldsymbol{E}^2} + r^{(\boldsymbol{E}\cdot\boldsymbol{B})}\delta_{\boldsymbol{E}\cdot\boldsymbol{B}}\,.$$

$$p = -\left(M^{-1}\right)_{12}d^{(\Phi)} - \left(M^{-1}\right)_{22}d^{(\delta\varphi)}\,,$$
$$q = -\left(M^{-1}\right)_{11}d^{(\Phi)} - \left(M^{-1}\right)_{21}d^{(\delta\varphi)}\,,$$
$$r^{(v)} = d^{(v)} - \left[\left(M^{-1}\right)_{11}c_1^{(v)} + \left(M^{-1}\right)_{12}c_2^{(v)}\right]d^{(\Phi)} - \left[\left(M^{-1}\right)_{21}c_1^{(v)} + \left(M^{-1}\right)_{22}c_2^{(v)}\right]d^{(\delta\varphi)}\,,$$
$$r^{(\rho)} = d^{(\rho)} - \left[\left(M^{-1}\right)_{11}c_1^{(\rho)} + \left(M^{-1}\right)_{12}c_2^{(\rho)}\right]d^{(\Phi)} - \left[\left(M^{-1}\right)_{21}c_1^{(\rho)} + \left(M^{-1}\right)_{22}c_2^{(\rho)}\right]d^{(\delta\varphi)}\,,$$
$$r_1^{(\pi)} = d_1^{(\pi)} - \left[\left(M^{-1}\right)_{11}c_1^{(\pi)} + \left(M^{-1}\right)_{12}c_2^{(\pi)}\right]d^{(\Phi)} - \left[\left(M^{-1}\right)_{21}c_1^{(\pi)} + \left(M^{-1}\right)_{22}c_2^{(\pi)}\right]d^{(\delta\varphi)}\,,$$
$$r_2^{(\pi)} = d_2^{(\pi)}\,, \qquad r^{(\boldsymbol{E}^2)} = d^{(\boldsymbol{E}^2)}\,, \qquad r^{(\boldsymbol{E}\cdot\boldsymbol{B})} = d^{(\boldsymbol{E}\cdot\boldsymbol{B})}\,.$$

- source terms could be considered as known from background

R. Durrer, R. von Eckardstein, DG, K. Schmitz, O. Sobol, and S. Vilchinskii, arXiv:2404.19694

two-point and three-point functions

$$\zeta_p(\boldsymbol{k},\eta) \;=\; \int_{-\infty}^{\eta} d\eta'\, G_{\boldsymbol{k}}(\eta,\eta')\mathcal{S}_{\boldsymbol{k}}(\eta')$$

$$\langle \zeta_{\boldsymbol{k}}(\eta)\zeta_{\boldsymbol{k}'}(\eta)\rangle = \langle \zeta^{(0)}_{\boldsymbol{k}}(\eta)\zeta^{(0)}_{\boldsymbol{k}'}(\eta)\rangle + \int_{-\infty}^{\eta} d\eta' \int_{-\infty}^{\eta} d\eta''\, G_{\boldsymbol{k}}(\eta,\eta')G_{\boldsymbol{k}'}(\eta,\eta'')\langle \mathcal{S}_{\boldsymbol{k}}(\eta')\mathcal{S}_{\boldsymbol{k}'}(\eta'')\rangle\,.$$

$$\langle \zeta_{\boldsymbol{k}_1}(\eta)\zeta_{\boldsymbol{k}_2}(\eta)\zeta_{\boldsymbol{k}_3}(\eta)\rangle = \int_{-\infty}^{\eta} d\eta_1 \int_{-\infty}^{\eta} d\eta_2 \int_{-\infty}^{\eta} d\eta_3\, G_{\boldsymbol{k}_1}(\eta,\eta_1)G_{\boldsymbol{k}_2}(\eta,\eta_2)G_{\boldsymbol{k}_3}(\eta,\eta_3)\langle \mathcal{S}_{\boldsymbol{k}_1}(\eta_1)\mathcal{S}_{\boldsymbol{k}_2}(\eta_2)\mathcal{S}_{\boldsymbol{k}_3}(\eta_3)\rangle\,,$$

Fourier decomposition

$$\hat{\boldsymbol{A}}(\eta, \boldsymbol{x}) = \int \frac{d^3\boldsymbol{p}}{(2\pi)^{3/2}\sqrt{I_1(\phi_c)}} \sum_{\lambda=\pm} \left[\boldsymbol{\epsilon}_\lambda(\boldsymbol{p})\mathcal{A}_{\lambda,\boldsymbol{p}}(\eta)\hat{b}_{\lambda,\boldsymbol{p}} e^{i\boldsymbol{p}\cdot\boldsymbol{x}} + \boldsymbol{\epsilon}^*_\lambda(\boldsymbol{p})\mathcal{A}^*_{\lambda,\boldsymbol{p}}(\eta)\hat{b}^\dagger_{\lambda,\boldsymbol{p}} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}}\right],$$

$$\mathcal{S}_{\boldsymbol{k}}(\eta) = \int \frac{d^3\boldsymbol{p}}{(2\pi)^{3/2}} \sum_{\lambda,\lambda'=\pm} \Big[K_1(\lambda, \boldsymbol{p}; \lambda', \boldsymbol{k}-\boldsymbol{p}; \eta, \boldsymbol{k})\hat{b}_{\lambda,\boldsymbol{p}}\hat{b}_{\lambda',\boldsymbol{k}-\boldsymbol{p}} + K_2(\lambda, \boldsymbol{p}; \lambda', \boldsymbol{p}-\boldsymbol{k}; \eta, \boldsymbol{k})\hat{b}_{\lambda,\boldsymbol{p}}\hat{b}^\dagger_{\lambda',\boldsymbol{p}-\boldsymbol{k}}$$

$$+K_3(\lambda, \boldsymbol{p}; \lambda', \boldsymbol{k}+\boldsymbol{p}; \eta, \boldsymbol{k})\hat{b}^\dagger_{\lambda,\boldsymbol{p}}\hat{b}_{\lambda',\boldsymbol{k}+\boldsymbol{p}} + K_4(\lambda, \boldsymbol{p}; \lambda', -\boldsymbol{k}-\boldsymbol{p}; \eta, \boldsymbol{k})\hat{b}^\dagger_{\lambda,\boldsymbol{p}}\hat{b}^\dagger_{\lambda',-\boldsymbol{k}-\boldsymbol{p}}\Big],$$

Fourier decomposition

$$\langle \mathcal{S}_{\boldsymbol{k}}(\eta')\mathcal{S}_{\boldsymbol{k}'}(\eta'')\rangle = \delta(\boldsymbol{k}+\boldsymbol{k}')\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\sum_{\lambda,\lambda'=\pm} 2K_1(\lambda,\boldsymbol{p};\lambda',\boldsymbol{k}-\boldsymbol{p};\eta',\boldsymbol{k})K_1^*(\lambda,\boldsymbol{p};\lambda',\boldsymbol{k}-\boldsymbol{p};\eta'',\boldsymbol{k})\,,$$

$$\langle \mathcal{S}_{\boldsymbol{k}_1}(\eta_1)\mathcal{S}_{\boldsymbol{k}_2}(\eta_2)\mathcal{S}_{\boldsymbol{k}_3}(\eta_3)\rangle = \delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\int \frac{d^3\boldsymbol{p}}{(2\pi)^{9/2}}\sum_{\lambda_1,\lambda_2,\lambda_3=\pm} 8K_1(\lambda_1,\boldsymbol{p};\lambda_2,\boldsymbol{k}_1-\boldsymbol{p};\eta_1,\boldsymbol{k}_1)$$
$$\times K_3(\lambda_2,\boldsymbol{k}_1-\boldsymbol{p};\lambda_3,-\boldsymbol{k}_3-\boldsymbol{p};\eta_2,\boldsymbol{k}_2)K_1^*(\lambda_3,-\boldsymbol{k}_3-\boldsymbol{p};\lambda_1,\boldsymbol{p};\eta_3,\boldsymbol{k}_3)$$

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the model

- assumptions

$$I_1(\phi_c) \equiv 1, \qquad I_1'(\phi_c) = 0, \qquad I_2(\phi_c) = \frac{\alpha}{f}\phi_c, \qquad I_2'(\phi_c) = \frac{\alpha}{f} = \text{const}$$

$$\langle \boldsymbol{E}^2 \rangle = \langle \boldsymbol{B}^2 \rangle = \langle \boldsymbol{E} \cdot \boldsymbol{B} \rangle = 0$$

$$\xi = \frac{\alpha \dot{\phi}_c}{2Hf} = \text{const}, \qquad a(\eta) = -\frac{1}{H\eta}, \qquad -k\eta \to 0$$

$$\zeta'' + \frac{2z'}{z}\zeta' + k^2\zeta = \mathcal{S}_{\boldsymbol{k}}(\eta)$$

⬇

Green's function in terms of Hankel functions

simplified source terms

$$\zeta'' + 2\left(\frac{\phi_c''}{\phi_c'} - \frac{\mathcal{H}'}{\mathcal{H}} + \mathcal{H}\right)\zeta' + k^2\zeta = r^{(v)}F_v + r^{(\rho)}F_\rho + r_1^{(\pi)}F_\pi + r_2^{(\pi)}F_\pi' + r^{(\boldsymbol{E}^2)}\delta_{\boldsymbol{E}^2} + r^{(\boldsymbol{E}\cdot\boldsymbol{B})}\delta_{\boldsymbol{E}\cdot\boldsymbol{B}}\,,$$

$$\begin{aligned}
r^{(v)} &= \frac{4M_{\mathrm{P}}^2\mathcal{H}}{\phi_c'^2}\left[\frac{k^2}{3} + \frac{a^2V'(\phi_c)}{\phi_c'}\left(2\mathcal{H} - \frac{\mathcal{H}'}{\mathcal{H}} - \frac{\phi_c''}{\phi_c'} + \frac{V''(\phi_c)\phi_c'}{V'(\phi_c)}\right)\right],\\
r^{(\rho)} &= \frac{4M_{\mathrm{P}}^2\mathcal{H}}{3\phi_c'^2}\left(-2\mathcal{H} + \frac{\mathcal{H}'}{\mathcal{H}} - \frac{\phi_c''}{\phi_c'}\right),\\
r_1^{(\pi)} &= \frac{4M_{\mathrm{P}}^2k^2\mathcal{H}}{3\phi_c'^2}\left(-\mathcal{H} + \frac{\mathcal{H}'}{\mathcal{H}} + 2\frac{\phi_c''}{\phi_c'}\right),\\
r_2^{(\pi)} &= -\frac{4M_{\mathrm{P}}^2k^2\mathcal{H}}{3\phi_c'^2},\\
r^{(\boldsymbol{E}^2)} &= 0\,,\\
r^{(\boldsymbol{E}\cdot\boldsymbol{B})} &= \frac{2}{3}a^2I_2'(\phi_c)\frac{\mathcal{H}}{\phi_c'}\,.
\end{aligned}$$

R. Durrer, R. von Eckardstein, DG, K. Schmitz, O. Sobol, and S. Vilchinskii, arXiv:2404.19694

gauge field modes

$$\mathcal{A}''_{\lambda,\boldsymbol{p}}(\eta) + \left[p^2 + \lambda p \frac{2\xi}{\eta}\right] \mathcal{A}_{\lambda,\boldsymbol{p}}(\eta) = 0$$

$$\mathcal{A}_{\lambda,\boldsymbol{p}}(\eta) = \frac{1}{\sqrt{2p}} \mathcal{W}_\lambda(-p\eta), \qquad \mathcal{A}'_{\lambda,\boldsymbol{p}}(\eta) = \sqrt{\frac{p}{2}} \mathcal{U}_\lambda(-p\eta)$$

$$\mathcal{W}_\lambda(\rho) = G_0(\lambda\xi, \rho) + iF_0(\lambda\xi, \rho), \qquad \mathcal{U}_\lambda(\rho) = -G'_0(\lambda\xi, \rho) - iF'_0(\lambda\xi, \rho)$$

power spectrum

$$P_\zeta^{(\mathrm{inv.\ dec.})}(k) = e^{4\pi\xi}\left(\frac{H^2}{2\pi|\dot{\phi}_c|}\right)^4 f_2(\xi)$$

$$f_2(\xi) = \frac{e^{-4\pi\xi}}{8\pi}\int d^3\boldsymbol{p}_* \sum_{\lambda,\lambda'=\pm}\left(1-\lambda\lambda'\frac{\boldsymbol{p}_*\cdot\boldsymbol{p}'_*}{p_* p'_*}\right)^2 p_* p'_* \left|\mathcal{I}(\lambda, p_*; \lambda', p'_*; 1)\right|^2$$

$$\mathcal{I}(\lambda, p_*; \lambda', p'_*; x) = \int_0^{y_{\mathrm{UV}}} dy\,(\sin y - y\cos y)\Big\{(\xi+\xi h_1 + y h_3)\lambda' \mathcal{U}_\lambda\left(\frac{p_*}{x}y\right)\mathcal{W}_{\lambda'}\left(\frac{p'_*}{x}y\right)$$
$$+(\xi+\xi h_1 - y h_3)\lambda\mathcal{W}_\lambda\left(\frac{p_*}{x}y\right)\mathcal{U}_{\lambda'}\left(\frac{p'_*}{x}y\right) - h_2\left[\mathcal{U}_\lambda\left(\frac{p_*}{x}y\right)\mathcal{U}_{\lambda'}\left(\frac{p'_*}{x}y\right)+\lambda\lambda'\mathcal{W}_\lambda\left(\frac{p_*}{x}y\right)\mathcal{W}_{\lambda'}\left(\frac{p'_*}{x}y\right)\right]\Big\}$$

only superhorizon modes considered

UV cutoff at 2ξ

$h_n \rightarrow$ the impact of metric perturbations

impact of metric perturbations

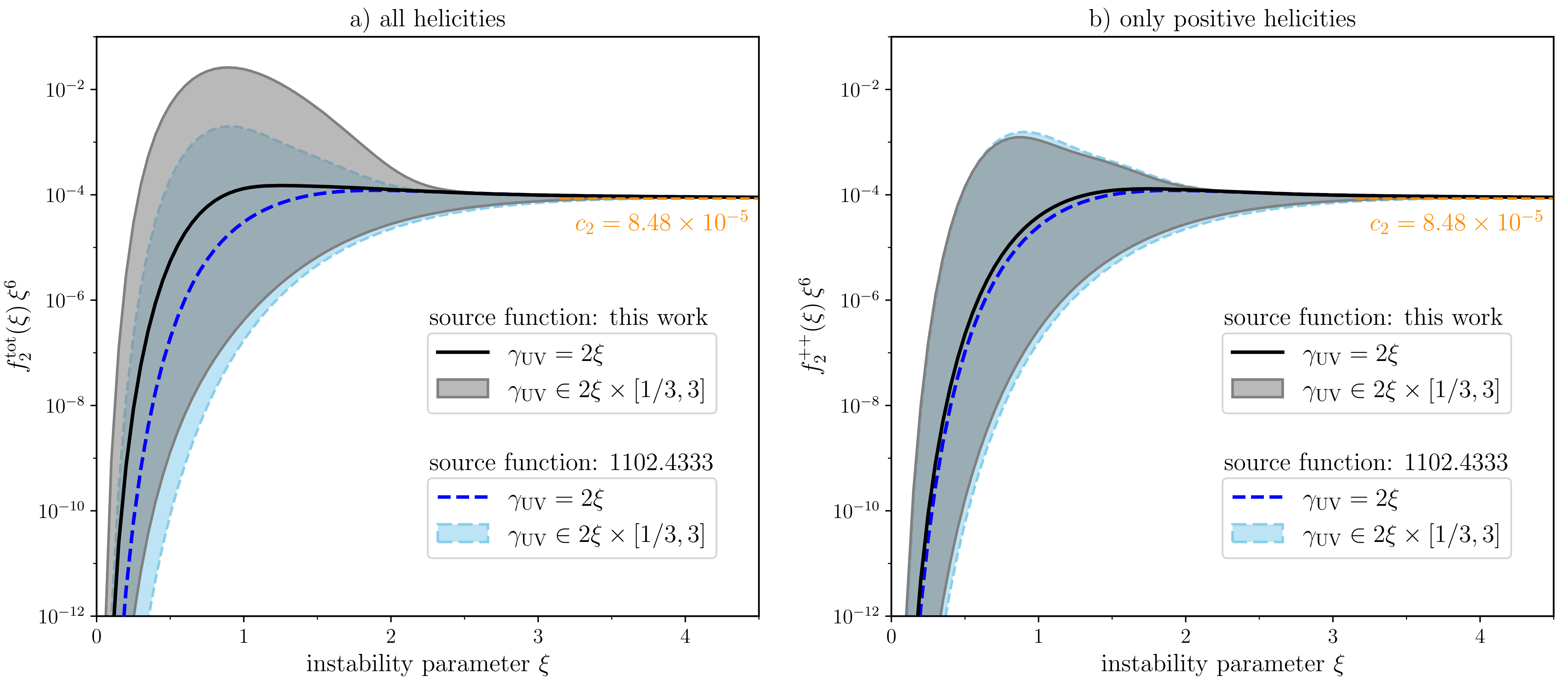
$$h_1(\eta) = -\frac{(\lambda p - \lambda' p')^2}{k^2}\,,$$

$$h_2(\eta) \simeq -2\frac{(\lambda p - \lambda' p')^2}{k^2} + \mathcal{O}(\epsilon)\,,$$

$$h_3(\eta) \simeq \frac{1}{2}\left[1 - \frac{(\lambda p - \lambda' p')^2}{k^2}\right]\frac{(\lambda p - \lambda' p')}{k} + \mathcal{O}(\epsilon, \eta_V)$$

- all 3 functions are proportional to $(\lambda p - \lambda' p') \xRightarrow{+ve\text{ pol.}} p - p'$

spectrum results



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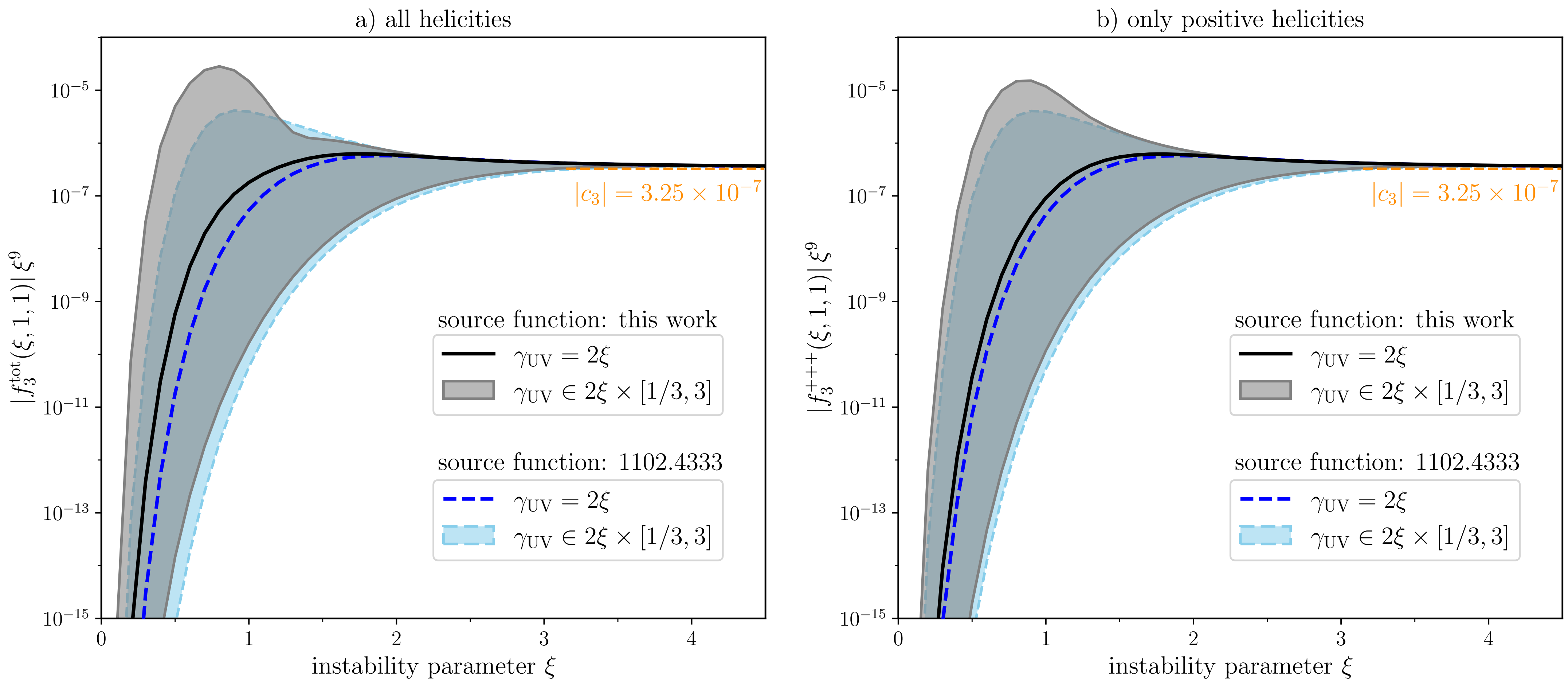
bispectrum

$$\mathscr{B}_\zeta(k_1, k_2, k_3) = \frac{3}{10}(2\pi)^{5/2}\left(\frac{H^2}{2\pi|\dot{\phi}|}\right)^6 \frac{e^{6\pi\xi}}{k^6}\frac{1 + x_2^3 + x_3^3}{x_2^3 x_3^3} f_3(\xi,\, x_2,\, x_3)$$

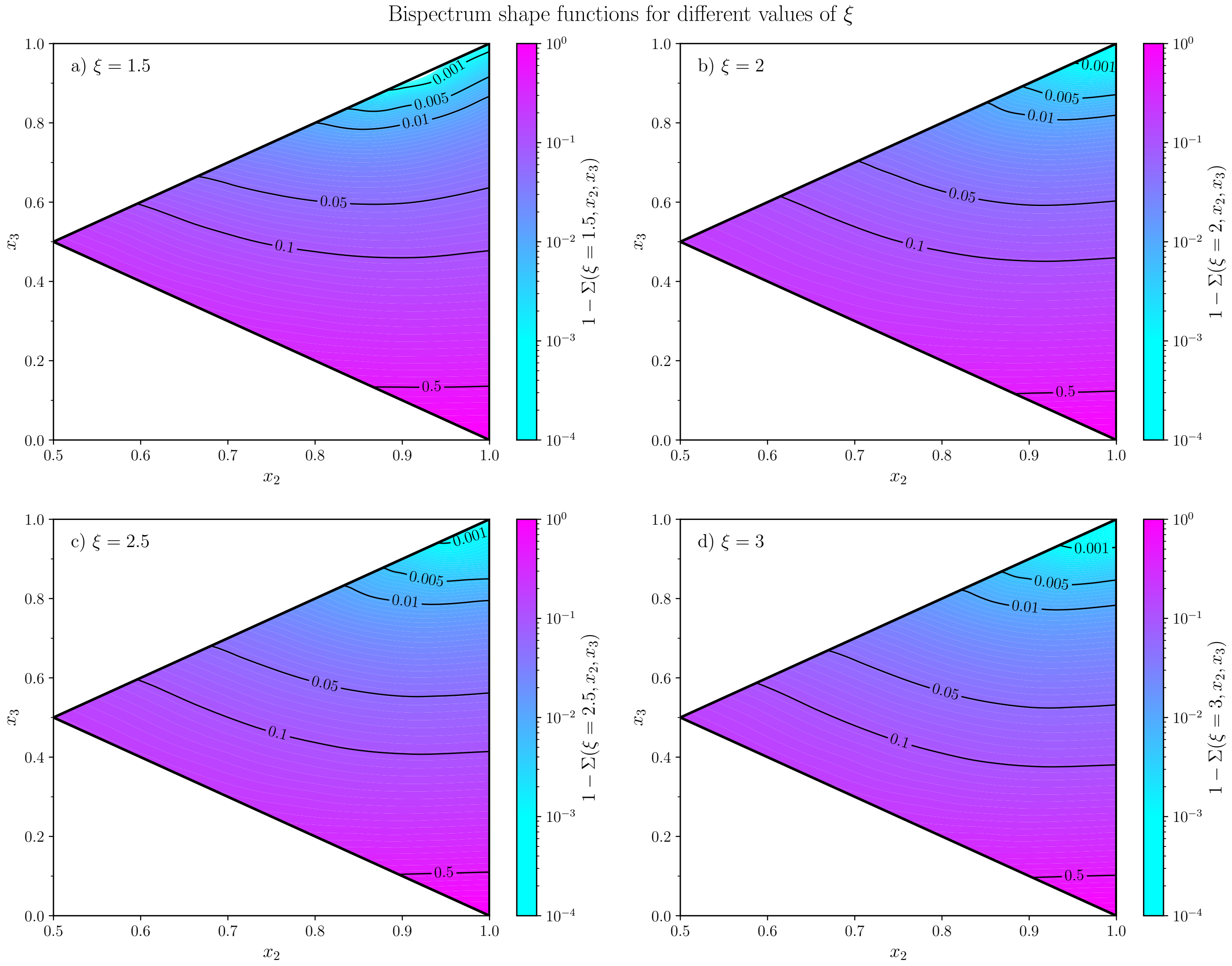
$$f_3(\xi, x_2, x_3) = -\frac{5}{3\pi}\frac{e^{-6\pi\xi}}{x_2 x_3(1 + x_2^3 + x_3^3)}\int d^3\boldsymbol{p}_* \sum_{\lambda_1,\lambda_2,\lambda_3=\pm} p_*|\boldsymbol{p}_* - \hat{\boldsymbol{k}}_1||\hat{\boldsymbol{k}}_3 x_3 + \boldsymbol{p}_*|$$
$$\times \left(\boldsymbol{\epsilon}_{\lambda_1}(\boldsymbol{p}_*) \cdot \boldsymbol{\epsilon}_{\lambda_2}(\hat{\boldsymbol{k}}_1 - \boldsymbol{p}_*)\right)\left(\boldsymbol{\epsilon}_{\lambda_2}(\boldsymbol{p}_* - \hat{\boldsymbol{k}}_1) \cdot \boldsymbol{\epsilon}_{\lambda_3}(-\hat{\boldsymbol{k}}_3 x_3 - \boldsymbol{p}_*)\right)\left(\boldsymbol{\epsilon}_{\lambda_3}(\hat{\boldsymbol{k}}_3 x_3 + \boldsymbol{p}_*)\right) \cdot \boldsymbol{\epsilon}_{\lambda_1}(-\boldsymbol{p}_*)\right)$$
$$\times \mathcal{I}\big(\lambda_1, p_*; \lambda_2, |\boldsymbol{p}_* - \hat{\boldsymbol{k}}_1|; 1\big)\mathcal{I}\big((\lambda_2, |\boldsymbol{p}_* - \hat{\boldsymbol{k}}_1|)^*; \lambda_3, |\hat{\boldsymbol{k}}_3 x_3 + \boldsymbol{p}_*|; x_2\big)\mathcal{I}\big((\lambda_3, |\hat{\boldsymbol{k}}_3 x_3 + \boldsymbol{p}_*|)^*; (\lambda_1, p_*)^*; x_3\big)$$

bispectrum results



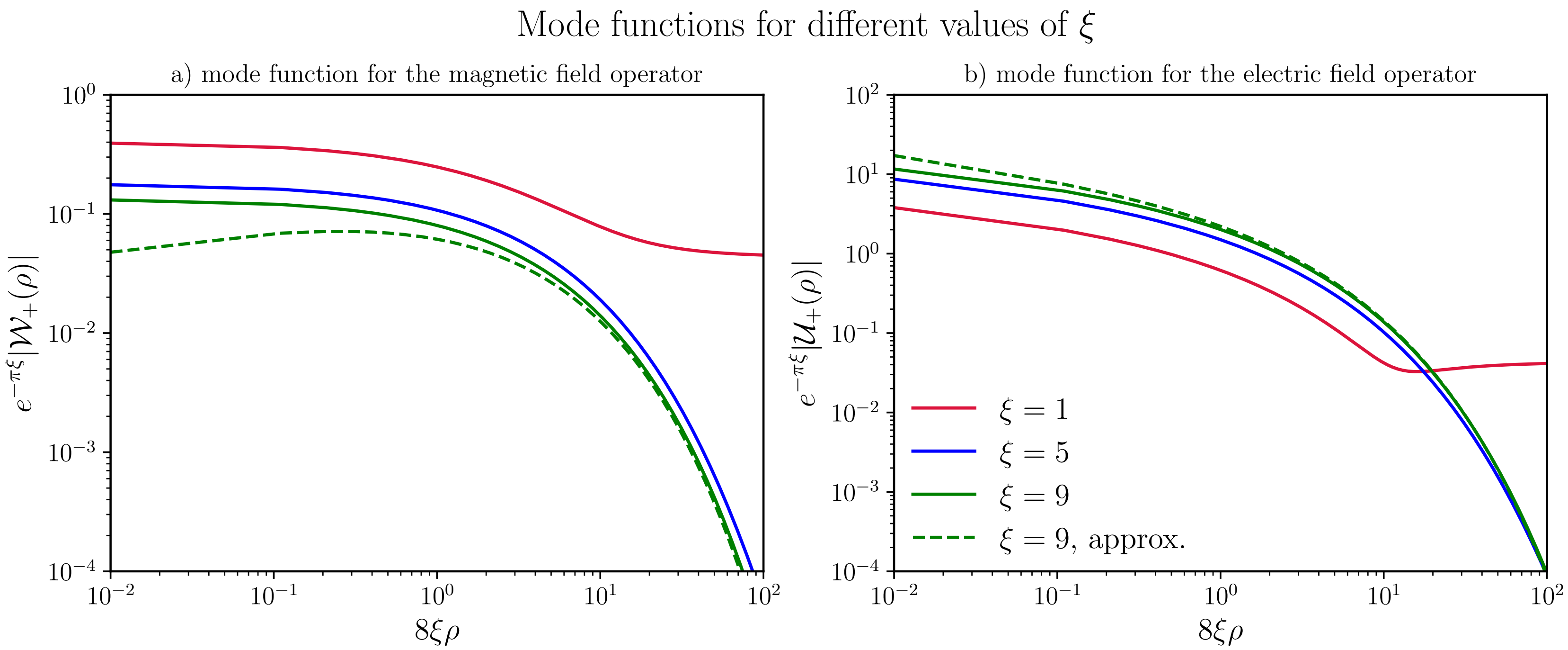
R. Durrer, R. von Eckardstein, D. Garg, K. Schmitz, O. Sobol, and S. Vilchinskii, arXiv:2404.19694

shape function: close to equilateral



why is there so little impact?

- smaller ξ leads to more overlap for different p and nonzero $p - p'$
- for small ξ, results very sensitive to UV cutoff, and interpreting –ve helicity gets complicated



- backreaction → h_n and gauge modes will be modified

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$\mathcal{O}(1)$ effects possible

(cf. Angelo's talk yesterday)

more analysis needed!

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summary

- gauge fields have recently been shown to impact inflaton evolution
- this requires a more careful analysis of their impact on scalar perturbations
- reinstating metric perturbations and retaining all the EM perturbations modifies the scalar perturbation equation
- a simpler case of no backreaction and only axial coupling is analyzed
  - no significant impact of retaining the additional terms, although results very sensitive to the UV cutoff for smaller ξ
- keeping backreaction could yield $\mathcal{O}(1)$ effects, necessitating further analysis

thank you!