

# PRESENTATION

## How's Schwinger effect affect primordial Magnetogenesis

Tomohiro Fujita  
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2204.01180 and 2206.12218  
with Kume (Padova), Mukaida (KEK), Tada (Nagoya)

3<sup>rd</sup>. May. 2024 @EPFL Bernoulli Center

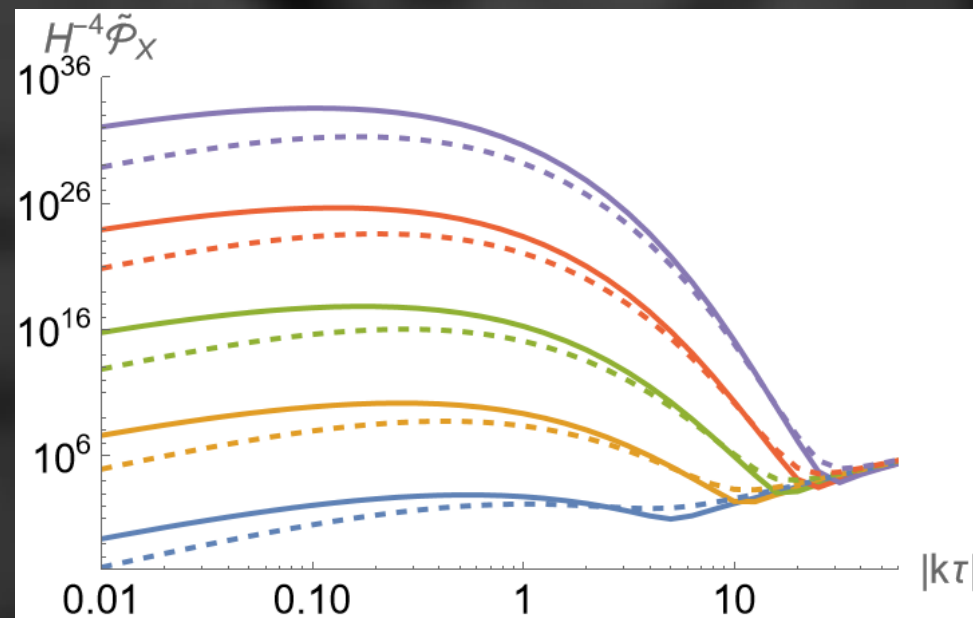
# MESSAGE



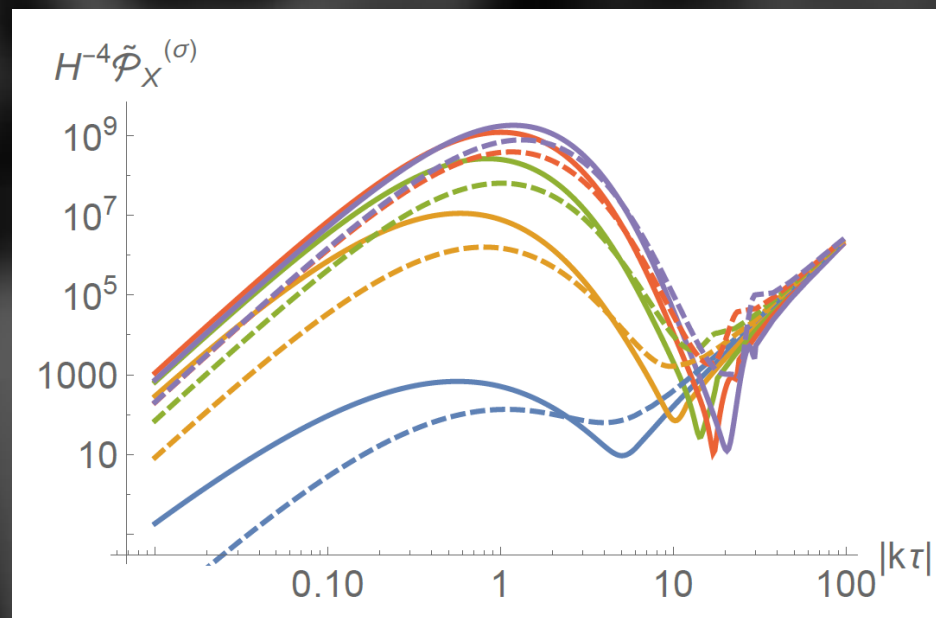
# MESSAGE

## 1. Schwinger effect matters

Without  $\Psi$



With  $\Psi$



# MESSAGE

## 2. Not an Ohmic conductivity

$$eJ_i \simeq \frac{e^3 B E_i}{6\pi^2 a^3 H} \coth\left(\frac{\pi B}{E}\right)$$



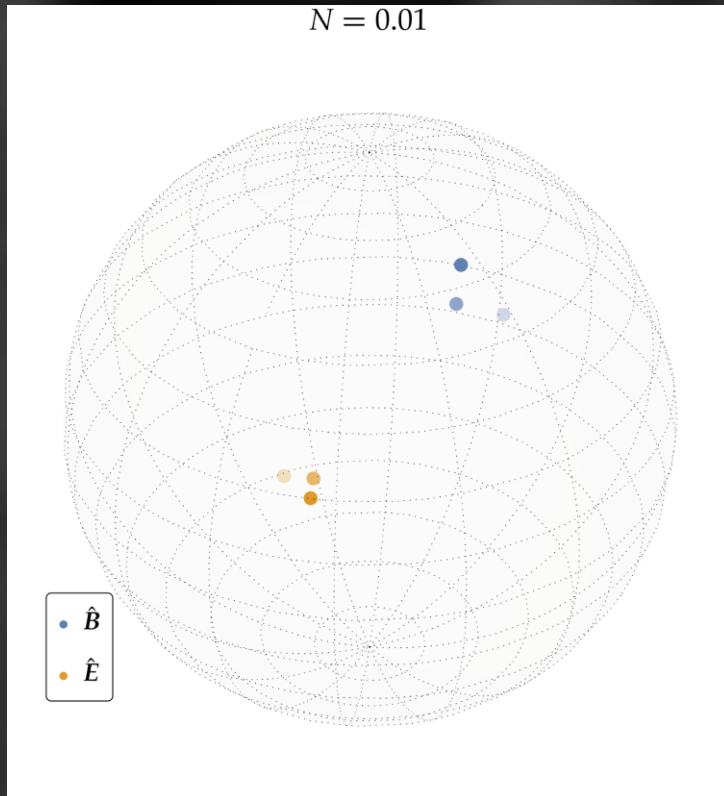
$$\delta J \simeq \sigma_E \delta E$$



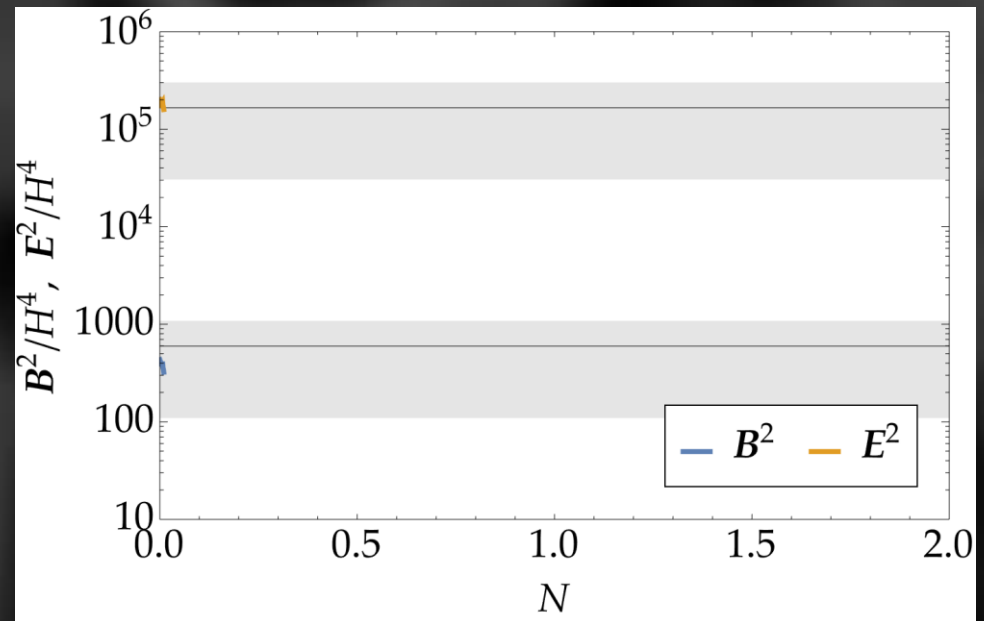
$$\delta J \simeq \sigma_E \delta E + \sigma_B \delta B$$

# MESSAGE

## 3. Future work: EMF dance



EMF Stochastic simulation  
without charged particles



# Plan of Talk

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1. Motivation
2. Review the case without  $\psi$
3. Solve the system of  $A$  &  $\psi$
4. Results
5. Summary

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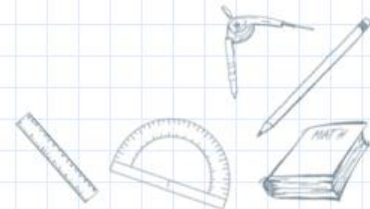


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**Setup** inflaton  $\phi$  – photon  $A_\mu$  – fermion  $\psi$  coupled system

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial\phi)^2 - V(\phi)}_{\text{Axionic inflaton}} - \underbrace{\frac{1}{4}FF - \frac{\alpha}{4f}\phi F\tilde{F}}_{\text{U(1) gauge field coupled to } \phi} + \underbrace{i\bar{\psi}\not{D}\psi}_{\text{Charged fermion}}$$





# 1 Motivation

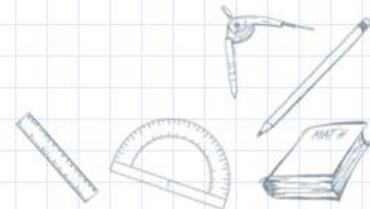


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## Motivations

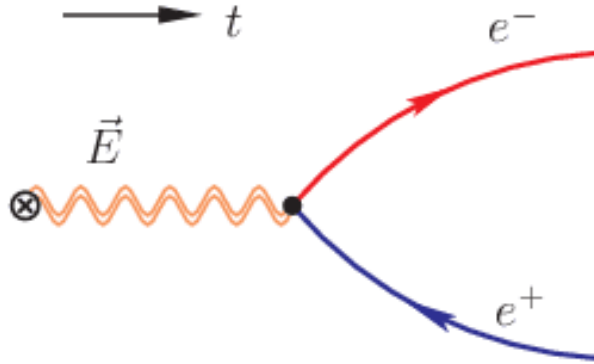
- ① Particle Physics: Shift symmetry of  $\phi$   $\longrightarrow$  Reheating requires coupling
- ② Phenomenology: Helical B  $\longrightarrow$  Baryogenesis & Magnetogenesis
- ③ Theory interest: Strong E  $\longrightarrow$  Schwinger effect



# 1 Motivation



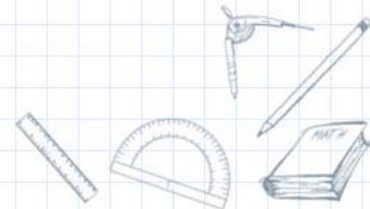
## Schwinger effect



Julian Schwinger(1918~1994)

- Sufficiently strong ( $eE > m^2$ ) electric field causes a **pair production** of charged particles. It's a **non-perturbative process** in QED.
- Not yet detected. It may be observed by EBI or X-FEL etc...
- In the early universe, however, It may have played an **important role**.

G. V. Dunne, Eur. Phys. J. D55, 327-340  
A. Ringwald, Phys. Lett. B510, 107-116



# Is Schwinger effect relevant?

$$a_{\text{end}} \simeq 10^{-33} \sqrt{M_{\text{Pl}}/H_{\text{inf}}}$$

Helicity  
conservation

$$\text{const} = a_{\text{end}}^3 \lambda_{\text{end}} B_{\text{end}}^2 = \lambda_0 B_0^2$$

Blazar  
Observation

$$\lambda_0 B_0^2 \gtrsim (10^{-17} \text{G})^2 \times 1 \text{Mpc}$$

Axion inflation

$$\lambda_{\text{end}} \sim H_{\text{inf}}^{-1}, \quad B_{\text{end}} \sim E_{\text{end}}$$

Fermion  
mass

$$m_\psi = y h \sim y H_{\text{inf}}$$



**Schwinger  
effect**

$$\frac{m_\psi^2}{e E_{\text{end}}} \sim 10^{-3} \frac{H_{\text{inf}}}{10^{13} \text{GeV}} \left( \frac{y}{2 \times 10^{-6}} \right)^2$$

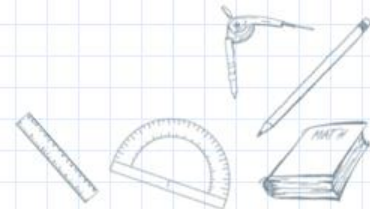
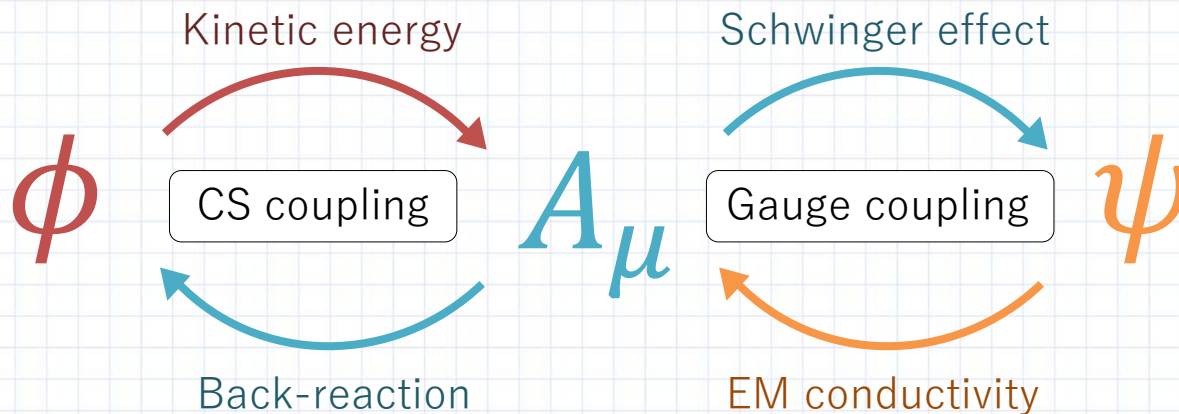
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## Interactions



# 1 Motivation

[Garretson+(1992), Field&Carroll(2000), Anber&Sorbo(2006)  
Durrer+(2011), Fujita+(2015), Adshead+(2016),... ]

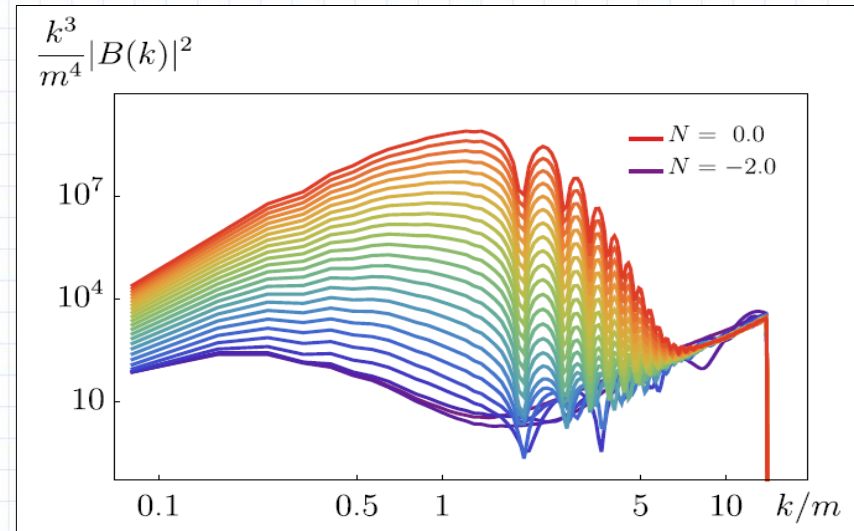


## Previous works

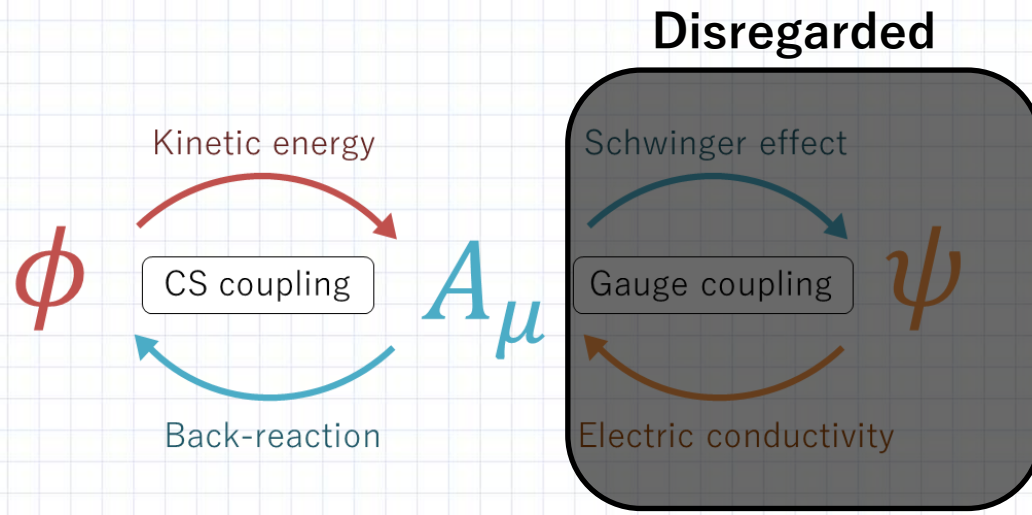
The  $\phi - A_\mu$  system well studied

➔  $A_\mu$  production at inf. end is dominant

However,  $\psi$  is not yet included!



[Lattice simulation by Cuissa & Figueroa (2018)]



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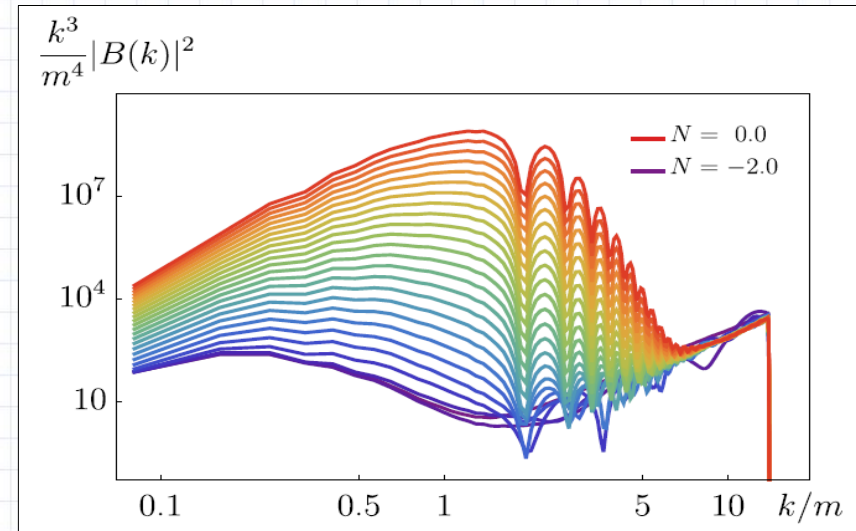


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## Difficulty

Non-linear & non-perturbative Dynamics

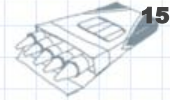
➔  $A(k), \psi(k)$ : different k-modes are coupled

➔ System is close to neither free mode nor thermal equilibrium

We need a new approach to solve it

[See also Domcke, Ema, Mukaida(2019);  
Gorbar, Schmitz, Sobol, Vilchinskii(2021)]

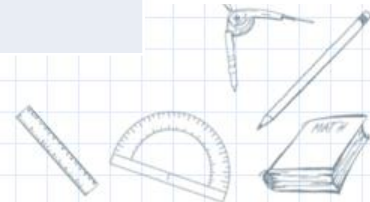




## Proposed Methods

cf. Valerie & Ruth's talk for Gradient Expansion

	Schwinger Effect	Evolution	Thermalization	Orientation	
Gradient Expansion	$\sigma_E$	Yes	No	No	Gorbar+(2021) Gorbar+(2022)
Mean-field Approx.	$\sigma_E, \sigma_B$	No	No	No	TF+(2022)
Kinetic	$\sigma_E$	Yes	Yes	No	Sobol+(2018) Gorbar+(2019) Okano&TF(2020)
Stochastic	No	Yes	No	Yes	TF+(2022)



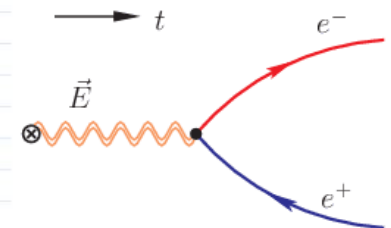
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## Short summary

- 1 (Axion inflaton + CS coupling) is interesting.  
Strong electromagnetic fields are produced.
- 2 Charged particles are hardly considered.  
But, Schwinger effect must produce them.
- 3 What happens then? How to analyze?  
We develop a new formalism.





# Plan of Talk

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**Assumption:** the inflaton rolls at a constant velocity  $\xi \equiv \frac{\alpha\dot{\phi}}{2fH}$

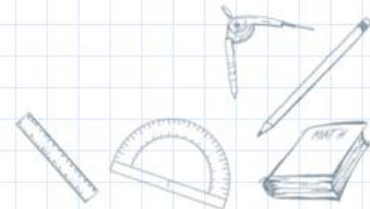
The EoM for the gauge field mode function  $\mathcal{A}_{\pm}$  is given by

$$\left[ \partial_{\tau}^2 + k^2 \boxed{\pm} 2k \frac{\xi}{\tau} \right] \mathcal{A}_{\pm}(\tau, k) = 0$$

Either  $\pm$  mode is amplified by the **tachyonic instability**.

In the slow-roll phase, an **analytic solution** is available.

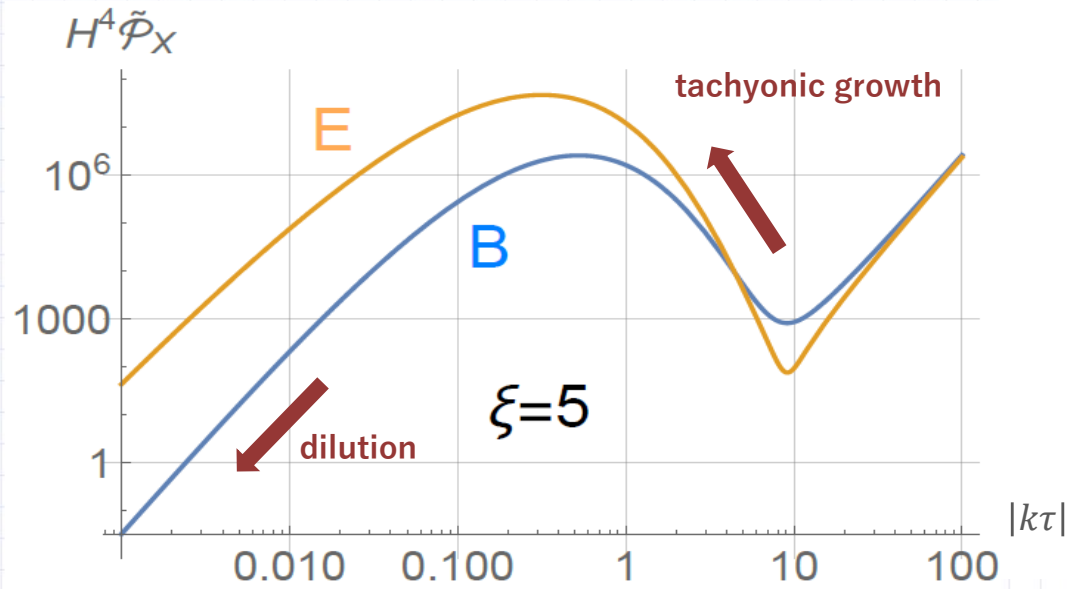
$$\text{If } \xi \equiv \frac{\alpha\dot{\phi}}{2fH} = \text{const.} > 0 \quad \longrightarrow \quad \mathcal{A}_{+} = \frac{1}{\sqrt{2k}} e^{\pi\xi/2} W_{-i\xi, 1/2}(2ik\tau)$$





## EM field production

(without  $\psi$ )



1. Due to the exp amplification, very strong EMFs are produced,  $E \gg B \gg H^2$ .
2. The typical EM length scale is  $L_{em} \simeq \xi/H$
3. The typical EM time scale is  $t_{em} \simeq 1/H$

$$\tilde{\mathcal{P}}_{BB}^+(\tau, k) = a^{-4} \mathcal{P}_{BB}^+(\tau, k) = \frac{k^5}{2\pi^2 a^4} |\mathcal{A}_+(\tau, k)|^2 = \frac{|k\tau|^4 H^4}{4\pi^2} e^{\pi\xi} |W(-k\tau)|^2, \quad \tilde{\mathcal{P}}_{EE}^+(\tau, k) = a^{-4} \mathcal{P}_{EE}^+(\tau, k) = \frac{k^3}{2\pi^2 a^4} |\partial_\tau \mathcal{A}_+(\tau, k)|^2 = \frac{|k\tau|^4 H^4}{4\pi^2} e^{\pi\xi} |W'(-k\tau)|^2,$$

# 2 Review no-charged-particle case

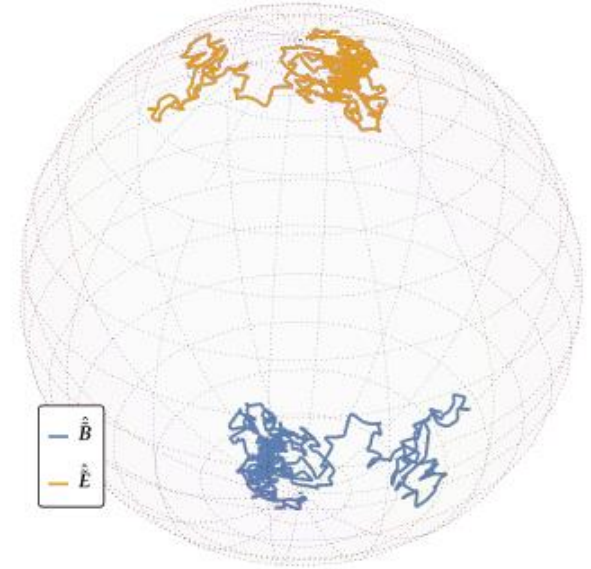


## EM field orientation

(without  $\psi$ )

Since the parity is fully violated,  
EM fields take an **anti-parallel** configuration.

$$\frac{\tilde{\mathcal{P}}_{BE}^+}{\sqrt{\tilde{\mathcal{P}}_{EE}^+ \tilde{\mathcal{P}}_{BB}^+}} \xrightarrow{|k\tau| \ll 2\xi} -1.$$



Evolution of  $\hat{E} \cdot \hat{B}$  for 0.5 e-folds

1. Due to the exp amplification, very strong EMFs are produced,  $E \gg B \gg H^2$ .
2. The typical EM length scale is  $L_{em} \simeq \xi/H$
3. The typical EM time scale is  $t_{em} \simeq 1/H$
4. E and B are anti-parallel,  $\hat{E} \cdot \hat{B} = -1$

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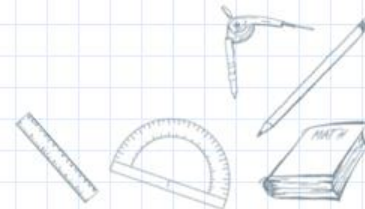
### 4 properties in the no charged particle case

1 Strong EMFs are produced:  $E, B \gg H^2$

2 The EM length scale  $L_{\text{em}} \simeq \xi/H$

3 The EM time scale is  $\tau_{\text{em}} \simeq 1/H$

4 EMFs are anti-parallel:  $\hat{\mathbf{E}} \cdot \hat{\mathbf{B}} = -1$



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Axionic inflaton

U(1) gauge field

Charged fermion

**Assumption:** the inflaton rolls at a constant velocity  $\xi \equiv \frac{\alpha\phi}{2fH}$

The EoMs for the gauge field and fermion are **coupled** and **non-linear**

$$\left[ \hat{\gamma}^\mu (\partial_\mu + igQ\hat{A}_\mu) + \frac{3}{2}aH\hat{\gamma}^0 \right] \hat{\psi} = 0$$

$$\partial_\tau^2 A_i - \partial_j^2 A_i + \frac{2\xi}{\tau} \epsilon_{ijkl} \partial_j A_l = a^2 e J_i \quad J^\mu = \bar{\psi} \gamma^\mu \psi$$

We cannot exactly solve them... Then, we introduce two prescriptions

- 1 **Integrating out  $\psi$ :** Reduce the coupled EoMs into a single non-linear eq.
- 2 **Mean-field approx:** linear eq. for perturbation and consistency eq.



# 3 Integrating out $\psi$



[Domcke&Mukaida(2018)]

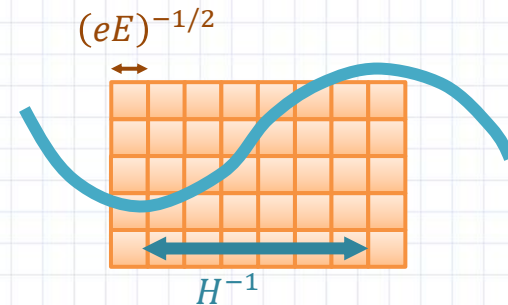
Remember the properties of the produced EMFs

- 1  $E, B \gg H^2$
- 2  $L_{\text{em}} \simeq \xi/H$
- 3  $\tau_{\text{em}} \simeq 1/H$

Typical momentum of the Schwinger produced fermion is  $\mathbf{p}_\psi \simeq \sqrt{e\mathbf{E}}$

Thus, a **hierarchy of scales** exists

$$L_\psi \sim t_\psi \sim (eE)^{-1/2} \ll L_{\text{em}} \sim t_{\text{em}} \sim H^{-1}$$

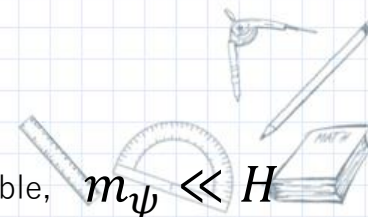


For fermions, EMFs look static and homogeneous,  $\tilde{\mathbf{E}}, \tilde{\mathbf{B}} \simeq \text{const.}$

Schwinger current induced by static, homogeneous & anti-parallel EMFs is known:

$$\partial_\tau (a^2 e J_i) = \frac{e^3 B E_i}{2\pi^2} \coth\left(\frac{\pi B}{E}\right).$$

NB; this current satisfies the chiral anomaly equation. Assumption: the fermion's mass is negligible,  $m_\psi \ll H$





### 3 Integrating out $\psi$



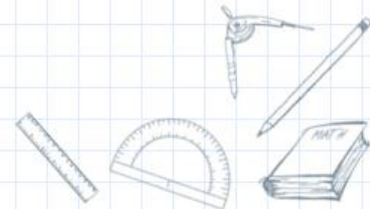
We need not  $\partial_\tau J_i$  but  $J_i$  itself.

**Assumption:** the physical EMFs are static,  $E, B \propto a^2$ , for  $t \gtrsim H^{-1}$

$$\partial_\tau(a^2 e J_i) = \frac{e^3 B E_i}{2\pi^2} \coth\left(\frac{\pi B}{E}\right).$$



$$e J_i \simeq \frac{e^3 B E_i}{6\pi^2 a^3 H} \coth\left(\frac{\pi B}{E}\right)$$



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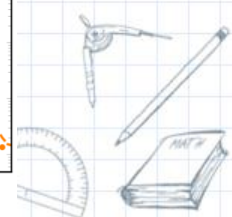
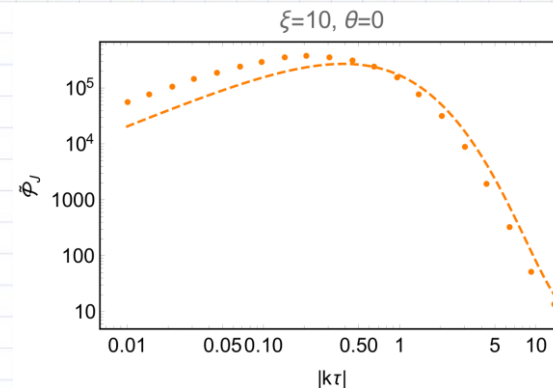
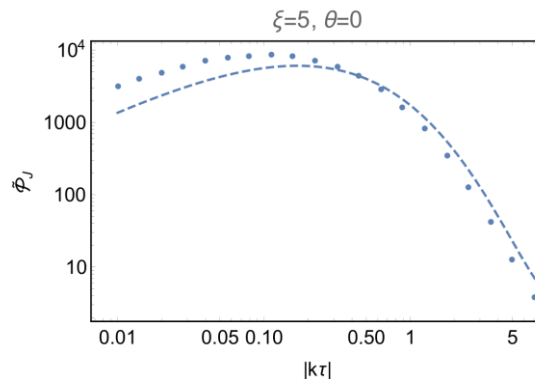
Since  $t_{em} \simeq H^{-1}$ , this expression may not be very accurate.

But on average, E and B amplitudes should be almost constant,

because the energy injection from the inflaton is constant,  $\xi = const.$

This assumption may lead to O(1) Error?

**Consistency Check:**  
Not bad in the growing phase



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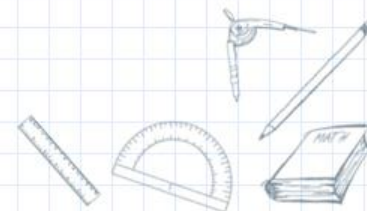
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We obtain a single **non-linear** EoM for A!!

$$\partial_\tau^2 A_i - \partial_j^2 A_i + \frac{2\xi}{\tau} \epsilon_{ijkl} \partial_j A_l = a^2 e J_i$$



### 3 Schwinger Conductivity?



Ohm's law??

**No!**  $J_i$  is not linear in  $E_i$

$$eJ_i \simeq \frac{e^3 B E_i}{6\pi^2 a^3 H} \coth\left(\frac{\pi B}{E}\right)$$

**None** of the followings is correct.

$$\mathbf{J}(t, \mathbf{x}) = \sigma_E(E_0, B_0)\mathbf{E}(t, \mathbf{x})$$



Effective Friction for **A**

$$\mathbf{J}(t, \mathbf{x}) = \sigma_B(E_0, B_0)\mathbf{B}(t, \mathbf{x})$$

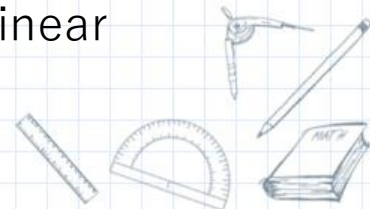


$\xi$  effectively decreases

$$\mathbf{J}(t, \mathbf{x}) = \sigma_{EB}B(t, \mathbf{x})\mathbf{E}(t, \mathbf{x})$$



Stay non-linear





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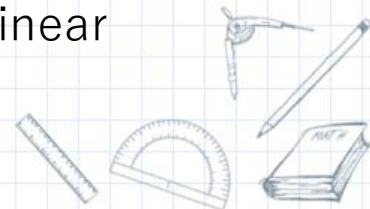
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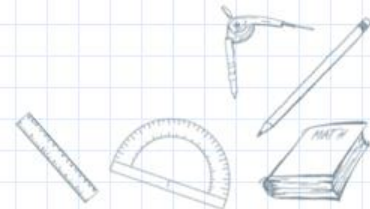


How to solve a full non-linear equation??

$$\partial_t^2 A_i - \partial_j^2 A_i + \frac{2\xi}{\tau} \epsilon_{ijkl} \partial_j A_l = a^2 e J_i$$

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We introduce **mean-field approximation**.





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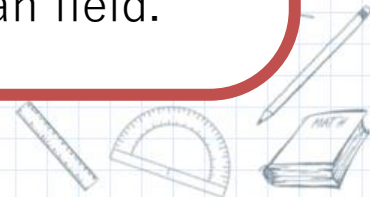
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We introduce **mean-field approximation**.

## Mean Field Approx

Focusing on one particle in a many-body system, we solve a one-body problem under the mean field created by the other particles.

Then, we also solve the self-consistency equation, whose averaged solution coincides with the mean field.





How to solve a full non-linear equation??

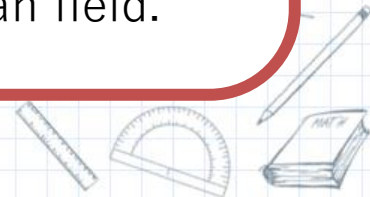
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Focusing on one particle in a many-body system, we solve a one-body problem under the mean field created by the other particles. **one Fourier mode**

Then, we also solve the self-consistency equation, whose averaged solution coincides with the mean field.







How to solve a full non-linear equation??

$$\partial_\tau^2 A_i - \partial_j^2 A_i + \frac{2\xi}{\tau} \epsilon_{ijkl} \partial_j A_l = a^2 e J_i \quad e J_i \simeq \frac{e^3 B E_i}{6\pi^2 a^3 H} \coth\left(\frac{\pi B}{E}\right)$$

We introduce **mean-field approx.** and split EMFs into a mean and a perturbation

$$\mathbf{E}(\tau, \mathbf{x}) \simeq \mathbf{E}_0 + \delta \mathbf{E}(\tau, \mathbf{x}), \quad \mathbf{B}(\tau, \mathbf{x}) \simeq \mathbf{B}_0 + \delta \mathbf{B}(\tau, \mathbf{x}).$$

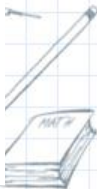
The Schwinger current is accordingly decomposed. ( $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{B}}_0 = -1$ , but  $\delta \mathbf{E} \cdot \delta \mathbf{B} \neq -1$ )

$$a^2 e \mathbf{J} = a^2 e (\mathbf{J}_0 + \delta \mathbf{J}),$$

$$a^2 e \mathbf{J}_0 = \frac{e^3 B_0 E_0}{6\pi^2 a H} \coth\left(\frac{\pi B_0}{E_0}\right) \mathbf{e}_z,$$

$$a^2 e \delta \mathbf{J} = \frac{e^3}{6\pi^2 a H} \left[ \left( \frac{B_0^3 \delta E_z - E_0^3 \delta B_z}{E_0^2 + B_0^2} \coth\left(\frac{\pi B_0}{E_0}\right) + (B_0 \delta E_z + E_0 \delta B_z) \frac{\pi B_0}{E_0} \operatorname{csch}^2\left(\frac{\pi B_0}{E_0}\right) \right) \mathbf{e}_z \right. \\ \left. + \frac{E_0^2 B_0 \delta \mathbf{E} - B_0^2 E_0 \delta \mathbf{B}}{E_0^2 + B_0^2} \coth\left(\frac{\pi B_0}{E_0}\right) \right].$$

$\delta \mathbf{E}, \delta \mathbf{B}$  may not  $\parallel \mathbf{E}_0, \mathbf{B}_0$





How to solve a full non-linear equation??

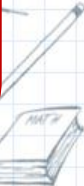
$$\partial_{\tau}^2 A_i - \partial_j^2 A_i + \frac{2\xi}{\tau} \epsilon_{ijkl} \partial_j A_l = a^2 e J_i \quad e J_i \simeq \frac{e^3 B E_i}{6\pi^2 a^3 H} \coth\left(\frac{\pi B}{E}\right)$$

We introduce **mean-field approx.** and split EMFs into a mean and a perturbation

$$\mathbf{E}(\tau, \mathbf{x}) \simeq \mathbf{E}_0 + \delta \mathbf{E}(\tau, \mathbf{x}), \quad \mathbf{B}(\tau, \mathbf{x}) \simeq \mathbf{B}_0 + \delta \mathbf{B}(\tau, \mathbf{x}).$$

The Schwinger current is accordingly decomposed. ( $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{B}}_0 = -1$ , but  $\delta \mathbf{E} \cdot \delta \mathbf{B} \neq -1$ )

$$\delta J \simeq \sigma_E \delta \mathbf{E} + \underline{\sigma_B \delta \mathbf{B}}$$





The EoM for the perturbation is

$$\left[ \partial_z^2 - \frac{\Sigma}{z} \partial_z + 1 - \frac{2\xi_{\text{eff}}}{z} \right] \mathcal{A}_+^{(\sigma)} = 0$$

$$z \equiv -k\tau$$

with **the electric and magnetic conductivity**:

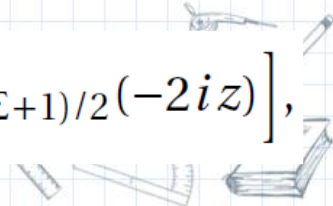
$$\Sigma \equiv \Sigma_E + \Sigma_{E'} \sin^2 \theta_k, \quad \xi_{\text{eff}} \equiv \xi - \frac{1}{2} (\Sigma_B + \Sigma_{B'} \sin^2 \theta_k) \quad \hat{E}_0 \cdot e^\pm(\hat{k}) = -\sin \theta_k / \sqrt{2}.$$

$$\Sigma_E \equiv \frac{e^3 B_0}{6\pi^2 a^2 H^2} \left( \frac{E_0^2}{E_0^2 + B_0^2} \coth \left( \frac{\pi B_0}{E_0} \right) \right), \quad \Sigma_{E'} \equiv \frac{e^3 B_0}{12\pi^2 a^2 H^2} \left( \frac{B_0^2}{E_0^2 + B_0^2} \coth \left( \frac{\pi B_0}{E_0} \right) + \frac{\pi B_0}{E_0} \text{csch}^2 \left( \frac{\pi B_0}{E_0} \right) \right)$$

$$\Sigma_B \equiv \frac{e^3 E_0}{6\pi^2 a^2 H^2} \left( \frac{B_0^2}{E_0^2 + B_0^2} \coth \left( \frac{\pi B_0}{E_0} \right) \right), \quad \Sigma_{B'} \equiv \frac{e^3 E_0}{12\pi^2 a^2 H^2} \left( \frac{E_0^2}{E_0^2 + B_0^2} \coth \left( \frac{\pi B_0}{E_0} \right) - \frac{\pi B_0}{E_0} \text{csch}^2 \left( \frac{\pi B_0}{E_0} \right) \right)$$

Fortunately, an analytic solution is available!

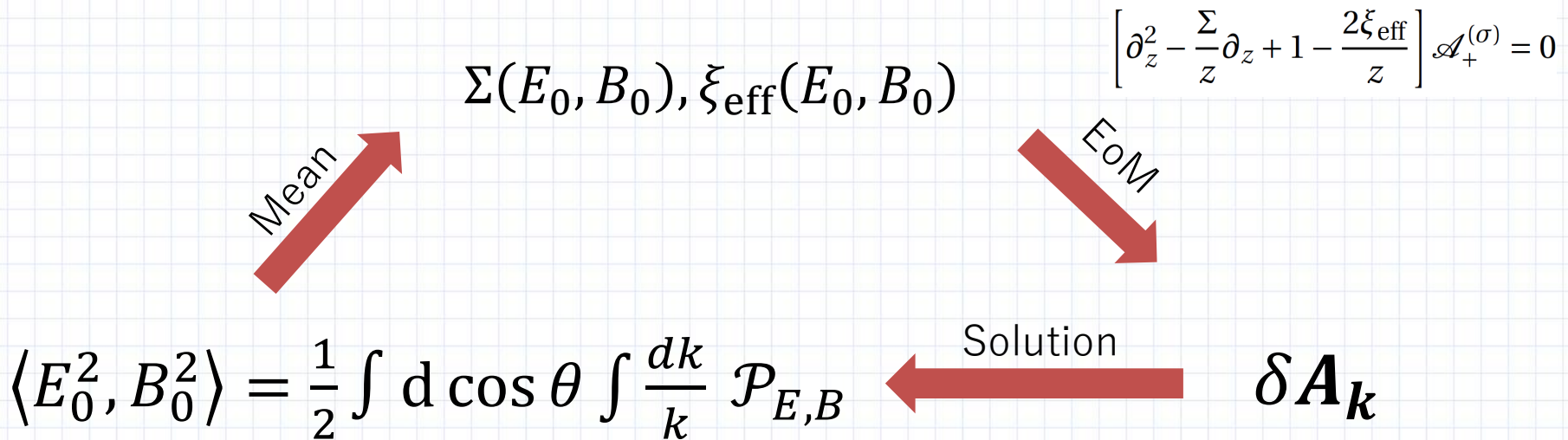
$$\mathcal{A}_+^{(\sigma)}(\tau, \mathbf{k}) = \frac{1}{\sqrt{2k}} e^{\pi \xi_{\text{eff}}/2} z^{\Sigma/2} \left[ c_1 W_{-i\xi_{\text{eff}}, (\Sigma+1)/2}(-2iz) + c_2 M_{-i\xi_{\text{eff}}, (\Sigma+1)/2}(-2iz) \right],$$



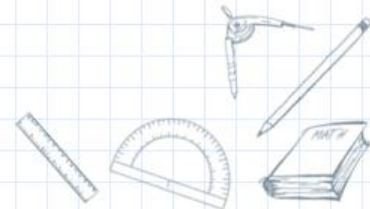


We impose the **consistency equation** to determine the mean-field value,

Require the integration over the perturbation reproduces the mean field amplitude



We numerically find self-consistent values of  $E_0$  and  $B_0$ ,  
under the assumption of  $\xi = \text{const.}$



# 3 Consistency equation



We impose the **consistency equation** to determine the mean-field value,  
Require the integration over the perturbation reproduces the mean field amplitude

Mean-field

Perturbation

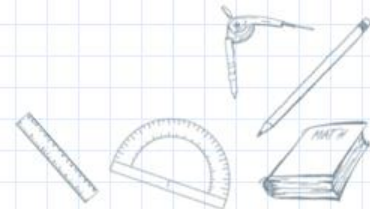
$$\tilde{E}_0 = \sqrt{2\rho_E(\tilde{E}_0, \tilde{B}_0)}, \quad \tilde{B}_0 = \sqrt{2\rho_B(\tilde{E}_0, \tilde{B}_0)},$$

$$\rho_B = \frac{1}{4} \int_{-1}^1 d\cos\theta \int_0^{2\xi} \frac{dz}{z} \tilde{\mathcal{P}}_{BB}^{+(\sigma)}(z, \theta), \quad \rho_E = \frac{1}{4} \int_{-1}^1 d\cos\theta \int_0^{2\xi} \frac{dz}{z} \tilde{\mathcal{P}}_{EE}^{+(\sigma)}(z, \theta),$$

$$\tilde{\mathcal{P}}_{BB}^{+(\sigma)}(z, \theta_k) = \frac{H^4}{4\pi^2} e^{\pi\xi_{\text{eff}}} z^{4+\Sigma} \left| c_1 W_\Sigma + c_2 M_\Sigma \right|^2, \quad \tilde{\mathcal{P}}_{EE}^{+(\sigma)}(z, \theta_k) = \frac{H^4}{4\pi^2} e^{\pi\xi_{\text{eff}}} z^{4+\Sigma} \left| c_1 W'_\Sigma + c_2 M'_\Sigma + \frac{\Sigma}{2z} (c_1 W_\Sigma + c_2 M_\Sigma) \right|^2,$$

We **numerically found** the consistent amplitudes of EMFs for given  $\xi$

(NB: In this iterative numerical process, we allowed **1% error**.)



# Plan of Talk

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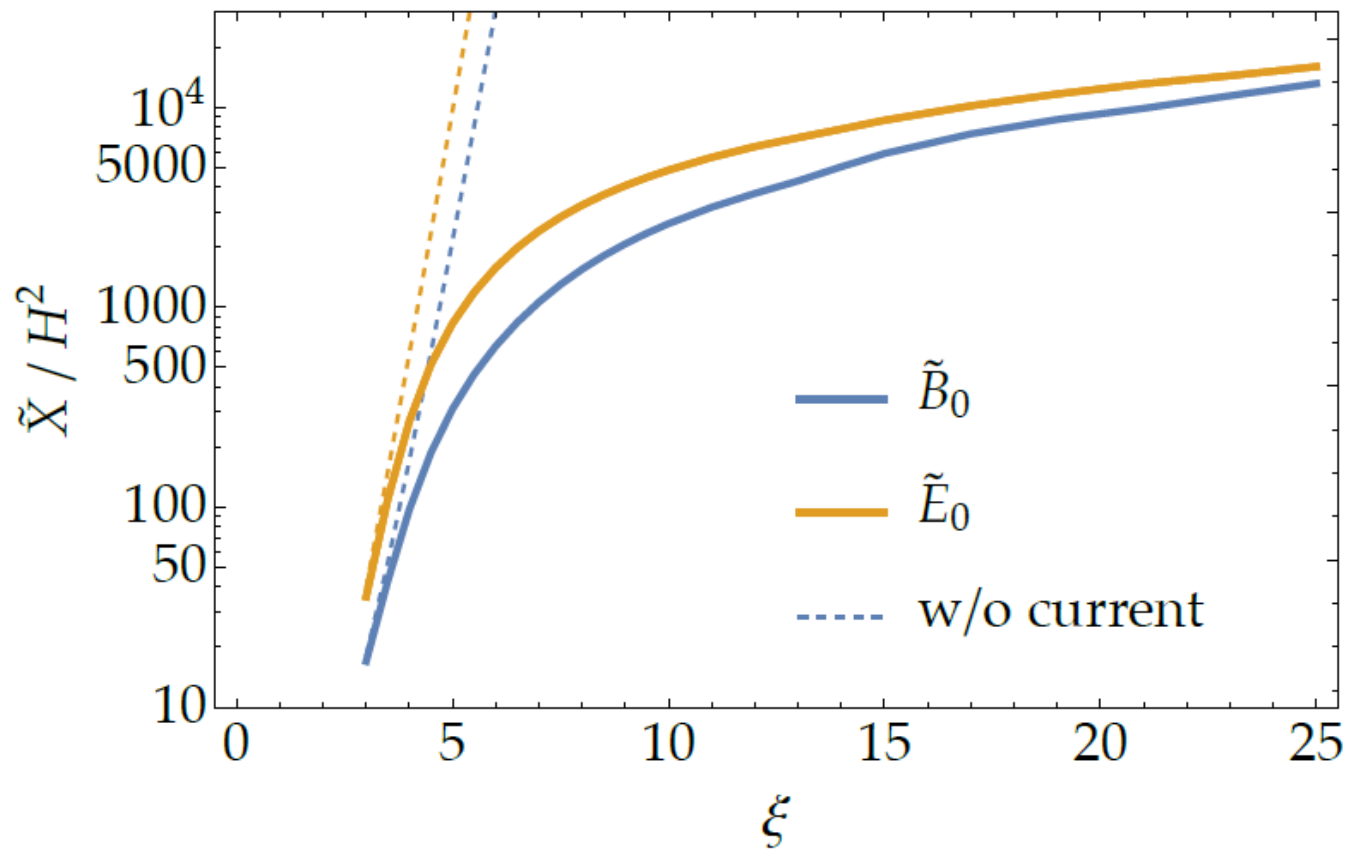


1. Motivation
2. Review the case without  $\psi$
3. Solve the system of A &  $\psi$
4. Results
5. Summary

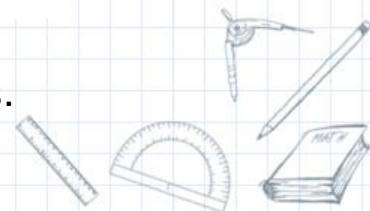
# 4 Numerical results



## Self-consistent mean-field amplitudes for EMFs



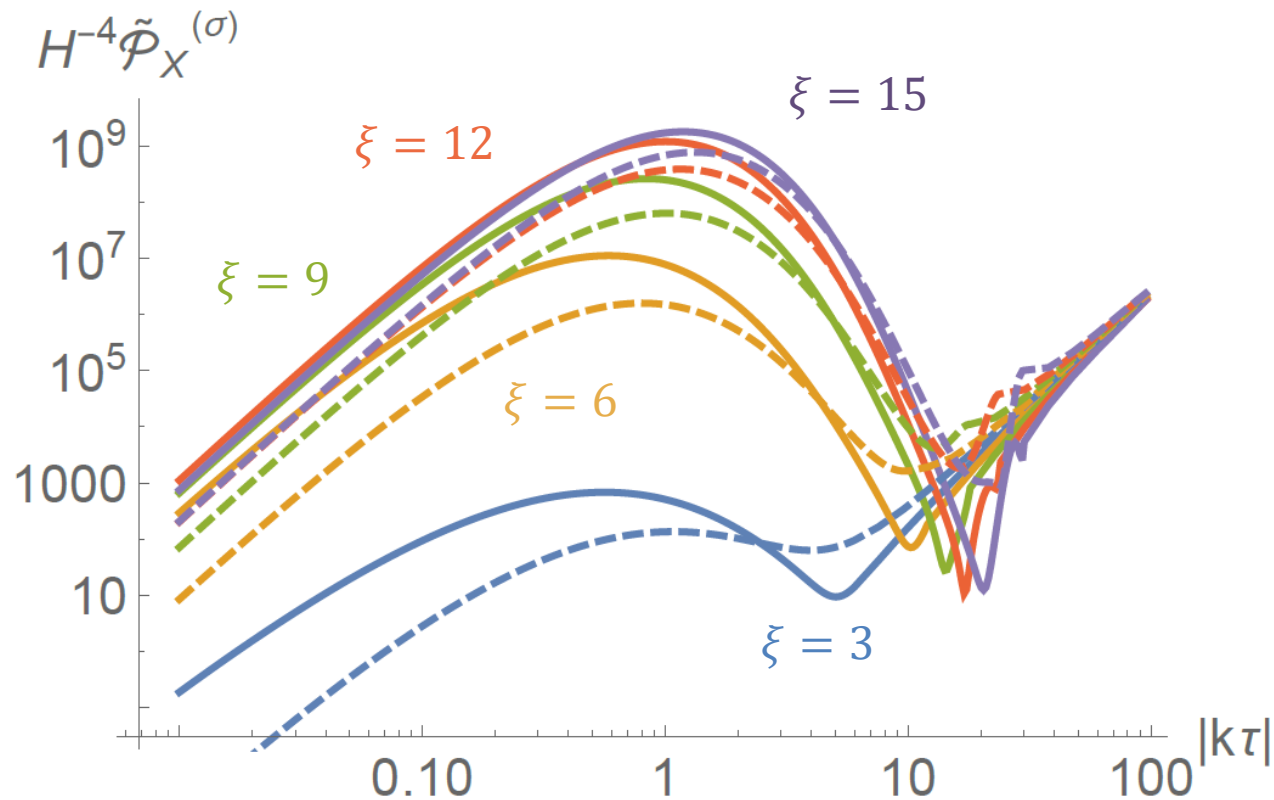
Charged fermions **drastically suppress** the EMF amplitudes.



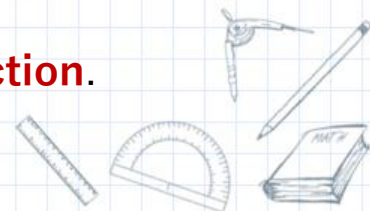
# 4 Numerical results



## E,B power spectra



- The spectra reach their peaks **earlier** due to the **effective friction**.
- EMFs keep the **4 properties**, which verifies our argument.

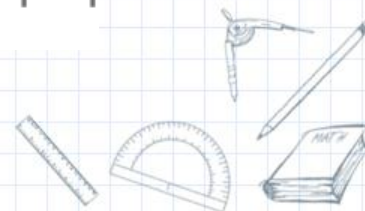
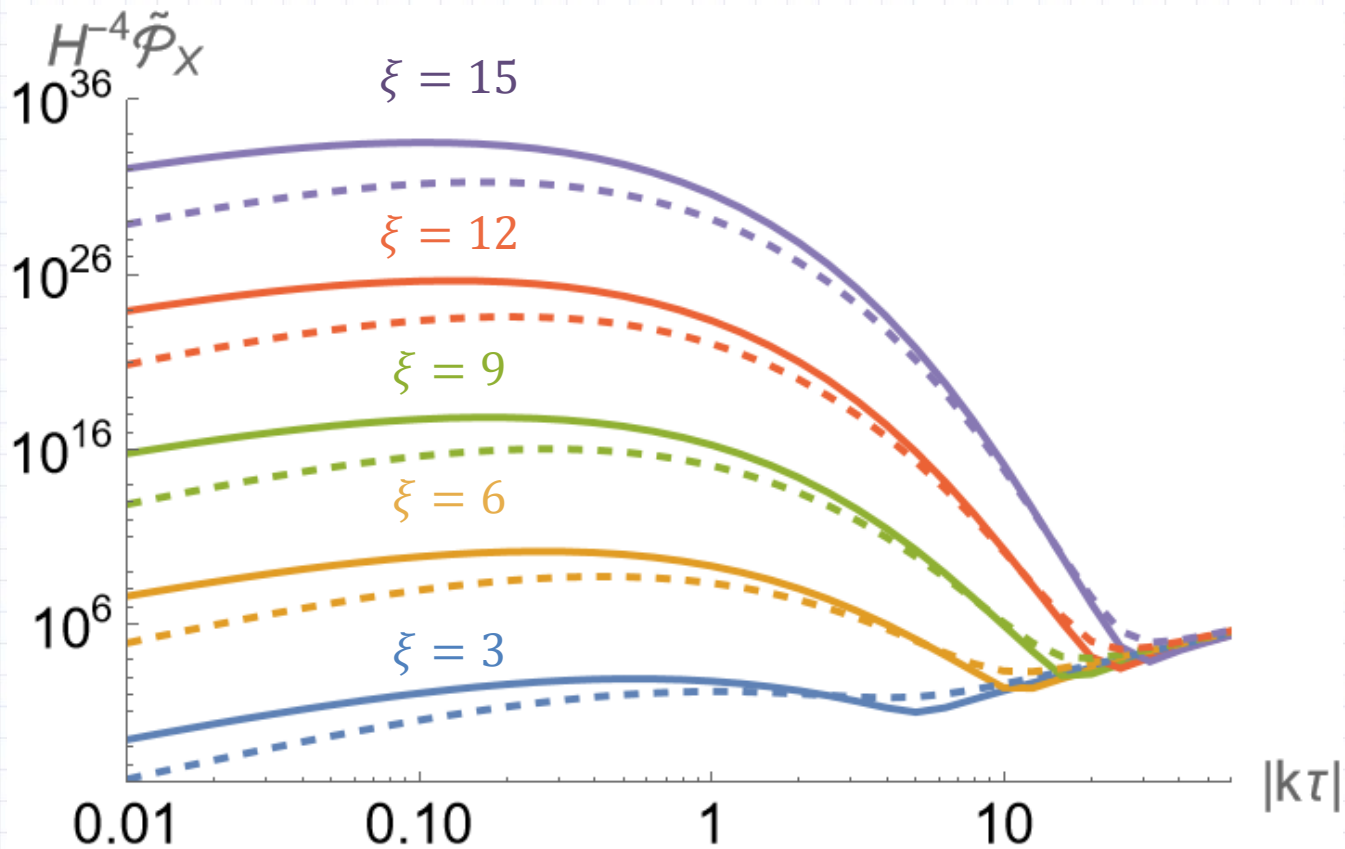


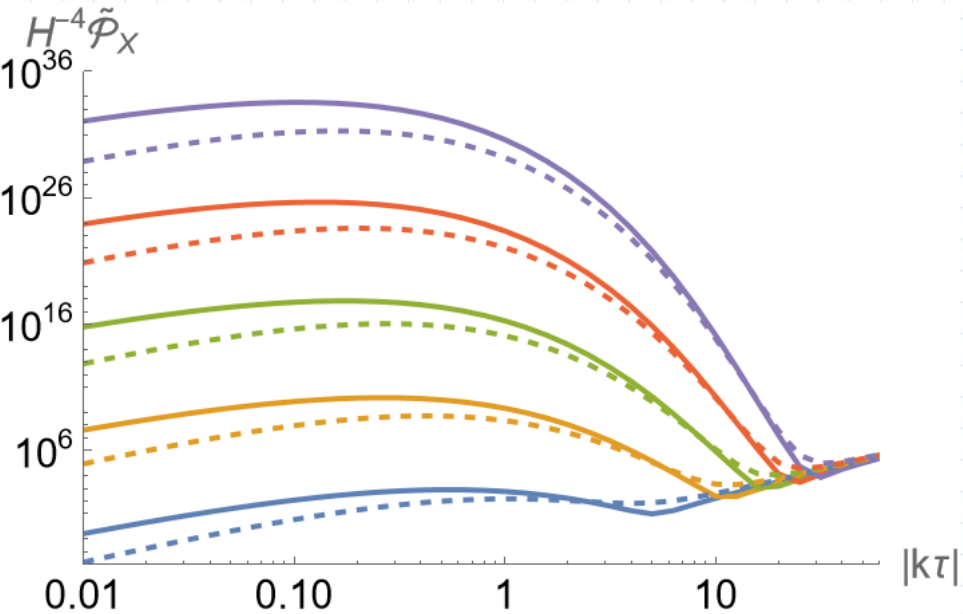
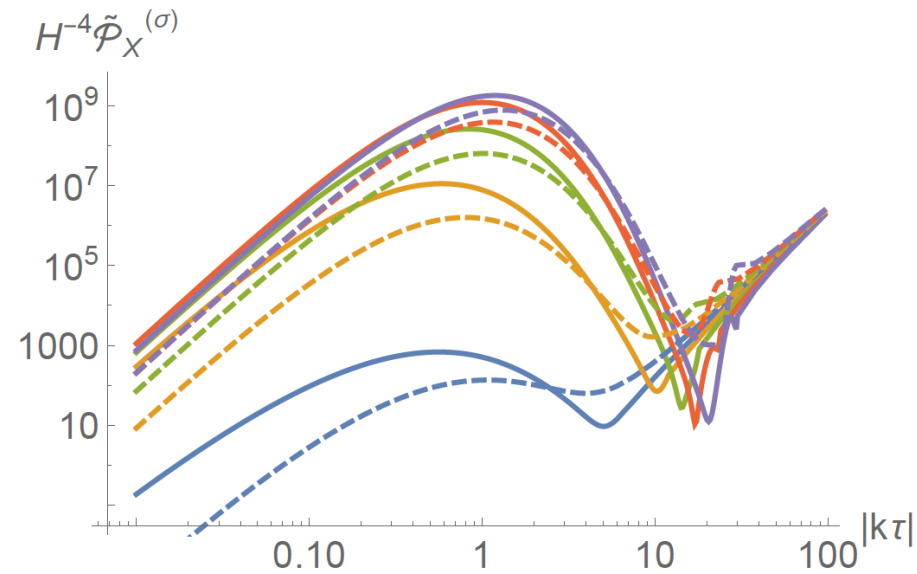


# 4 Numerical results

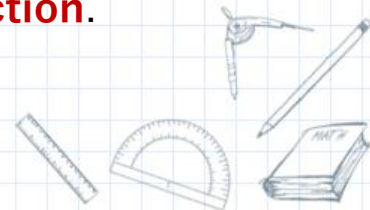


## No charged particle case



Without  $\Psi$ With  $\Psi$ 

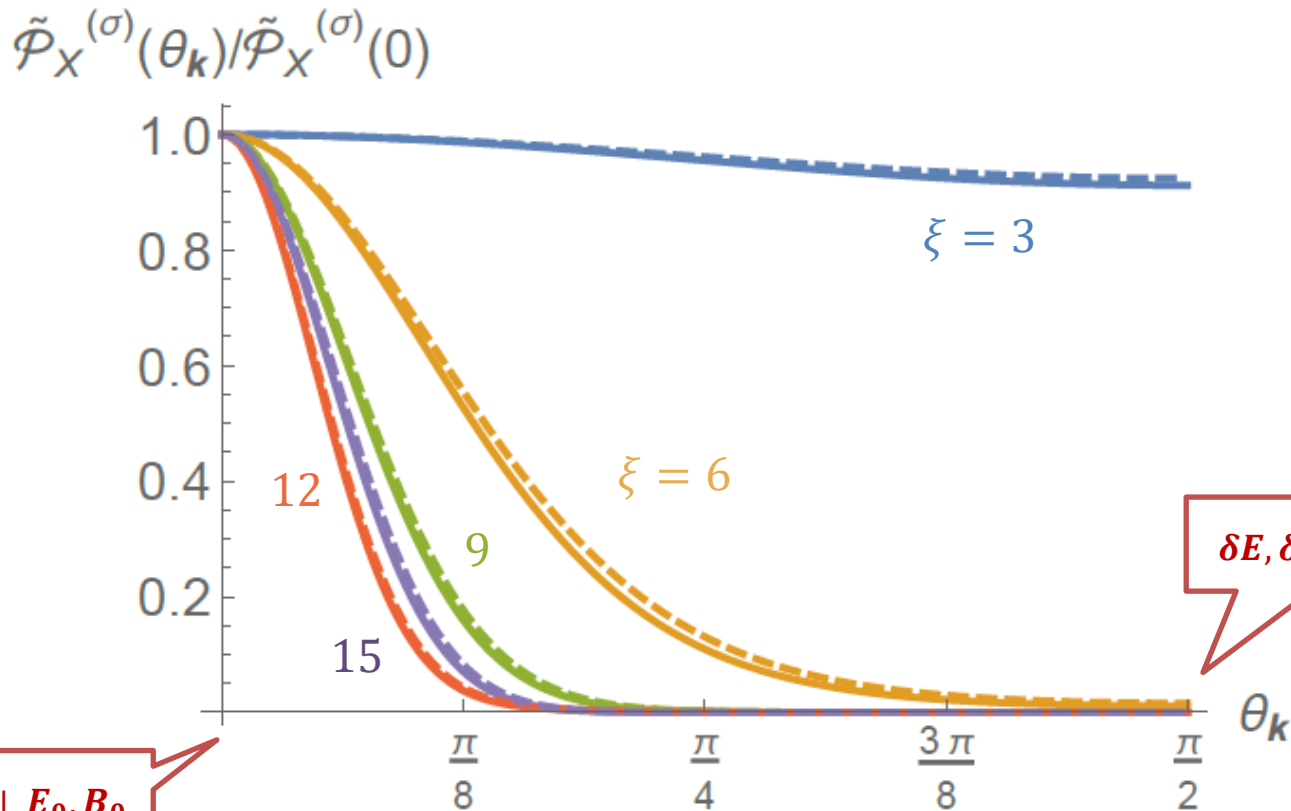
- The spectra reach their peaks **earlier** due to the **effective friction**.
- EMFs keep the **4 properties**, which verifies our argument.



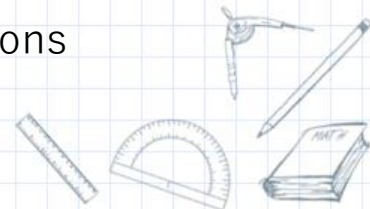
# 4 Numerical results



## Direction dependence of the power spectra



Schwinger current prevents the EMF production in similar directions  
**perpendicular** production is favored  $\Rightarrow$  **Rotation** of the EMFs??





The energy density of EMFs evolves as

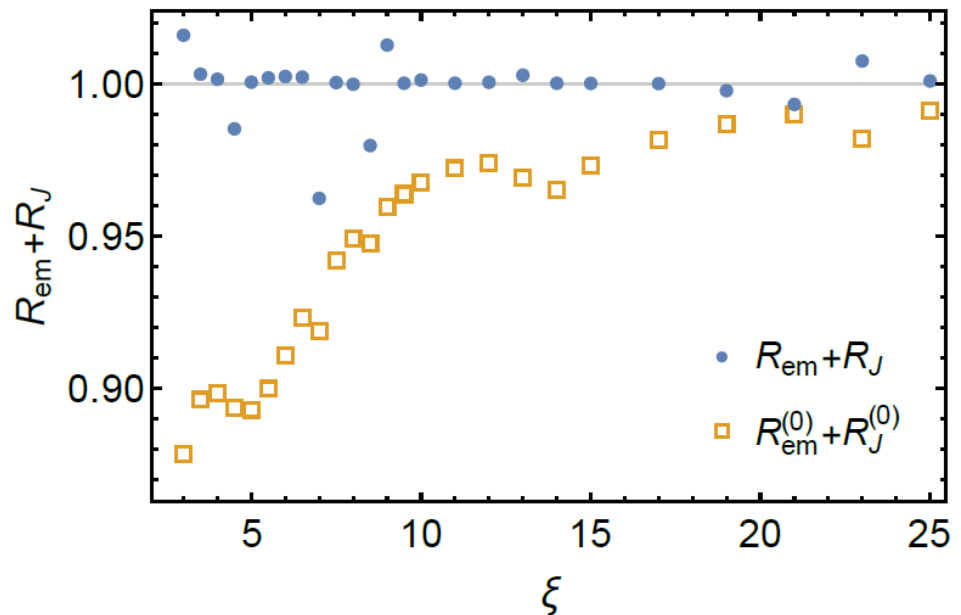
$$\langle \dot{\rho}_A \rangle = \underbrace{-2H \langle \tilde{\mathbf{E}}^2 + \tilde{\mathbf{B}}^2 \rangle}_{\text{Hubble dilution}} - \underbrace{2\xi H \langle \tilde{\mathbf{E}} \cdot \tilde{\mathbf{B}} \rangle}_{\text{Energy injection from } \phi} - \underbrace{e \langle \tilde{\mathbf{E}} \cdot \tilde{\mathbf{J}} \rangle}_{\text{produce \& accelerate charged fermions}},$$

Since we consider a **static** system,  $\langle \dot{\rho}_A \rangle$  should vanish and the 3 terms should be **balanced**.

$$R_{\text{em}} + R_J = 1,$$

$$R_{\text{em}} \equiv \frac{\langle \tilde{\mathbf{E}}^2 + \tilde{\mathbf{B}}^2 \rangle}{\xi |\langle \tilde{\mathbf{E}} \cdot \tilde{\mathbf{B}} \rangle|},$$

$$R_J \equiv \frac{e \langle \tilde{\mathbf{E}} \cdot \tilde{\mathbf{J}} \rangle}{2\xi H |\langle \tilde{\mathbf{E}} \cdot \tilde{\mathbf{B}} \rangle|}$$

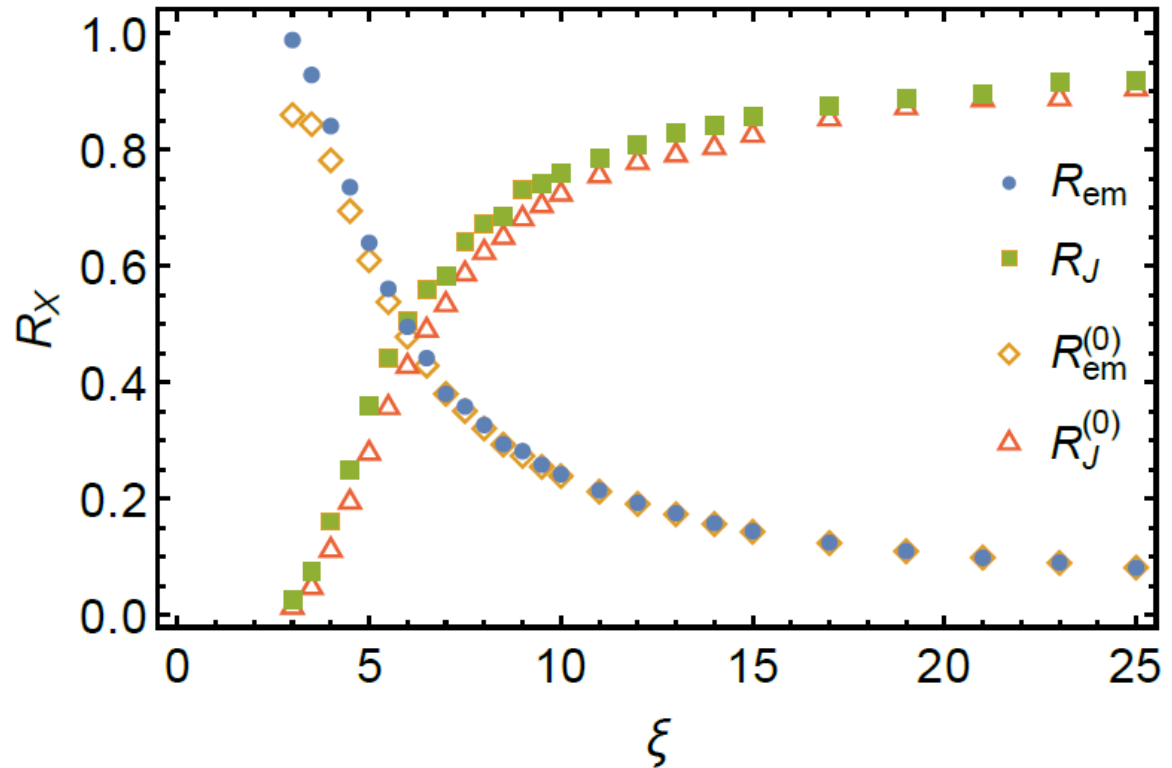




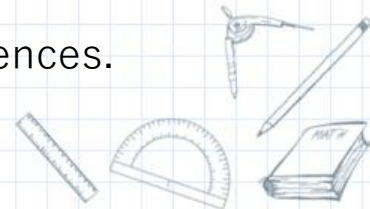
$$R_{\text{em}} + R_J = 1,$$

$$R_{\text{em}} \equiv \frac{\langle \tilde{\mathbf{E}}^2 + \tilde{\mathbf{B}}^2 \rangle}{\xi |\langle \tilde{\mathbf{E}} \cdot \tilde{\mathbf{B}} \rangle|},$$

$$R_J \equiv \frac{e \langle \tilde{\mathbf{E}} \cdot \tilde{\mathbf{J}} \rangle}{2\xi H |\langle \tilde{\mathbf{E}} \cdot \tilde{\mathbf{B}} \rangle|}$$



- For  $\xi \gtrsim 10$ , the energy transfer to the **fermions is dominant**.
- We don't know **why**... But it may have an interesting consequences.



# Plan of Talk

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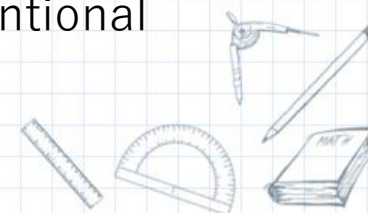
1. Motivation
2. Review the case without  $\psi$
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# SUMMARY



- Inflaton  $\phi$  – photon  $A_\mu$  – fermion  $\psi$  coupled system is well motivated but difficult. We need a **new approach** to solve this.
- We **integrated out**  $\psi$  by using the scale separation  $L_\psi \ll L_{em}$ , and introduced **mean-field approx.** to solve non-linear eq. for  $A_\mu$ . EM conductivities provide effective friction and reduction of  $\xi$ .
- We numerically solve the **consistency equation** to find the mean fields. The EM amplitudes are **drastically suppressed** compared to no- $\psi$  case.
- Interestingly, the **dominant part** of the injected energy from  $\phi$  goes to the charged fermions for  $\xi \gtrsim 10$ , which changes the conventional picture and may leads **new consequences**.





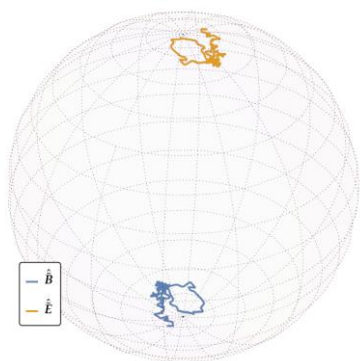
# Future Work



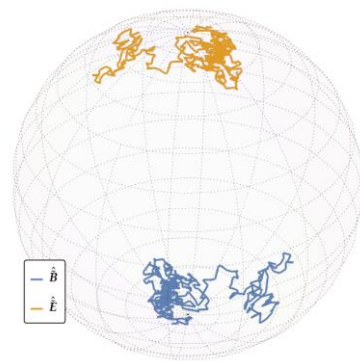
- Relax the  $\xi = \text{const.}$  assumption.  
Then we can explore the **inflation end** where  $\xi$  becomes maximum.
- Two unsatisfactory points of this work:
  1. Static EM assumption for  $t \gtrsim H^{-1}$
  2. Consistency eq. is imposed only on the EMF amplitude not direction.

➡ We cannot incorporate the **rotation** of EMFs

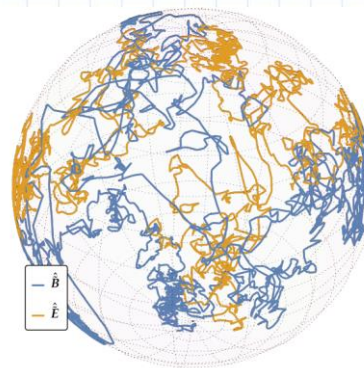
[TF, Mukaida, Tada, 2206.12218]



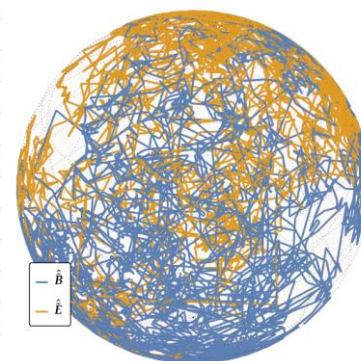
$\Delta N = 0.1$



$\Delta N = 0.5$



$\Delta N = 2$



$\Delta N = 10$





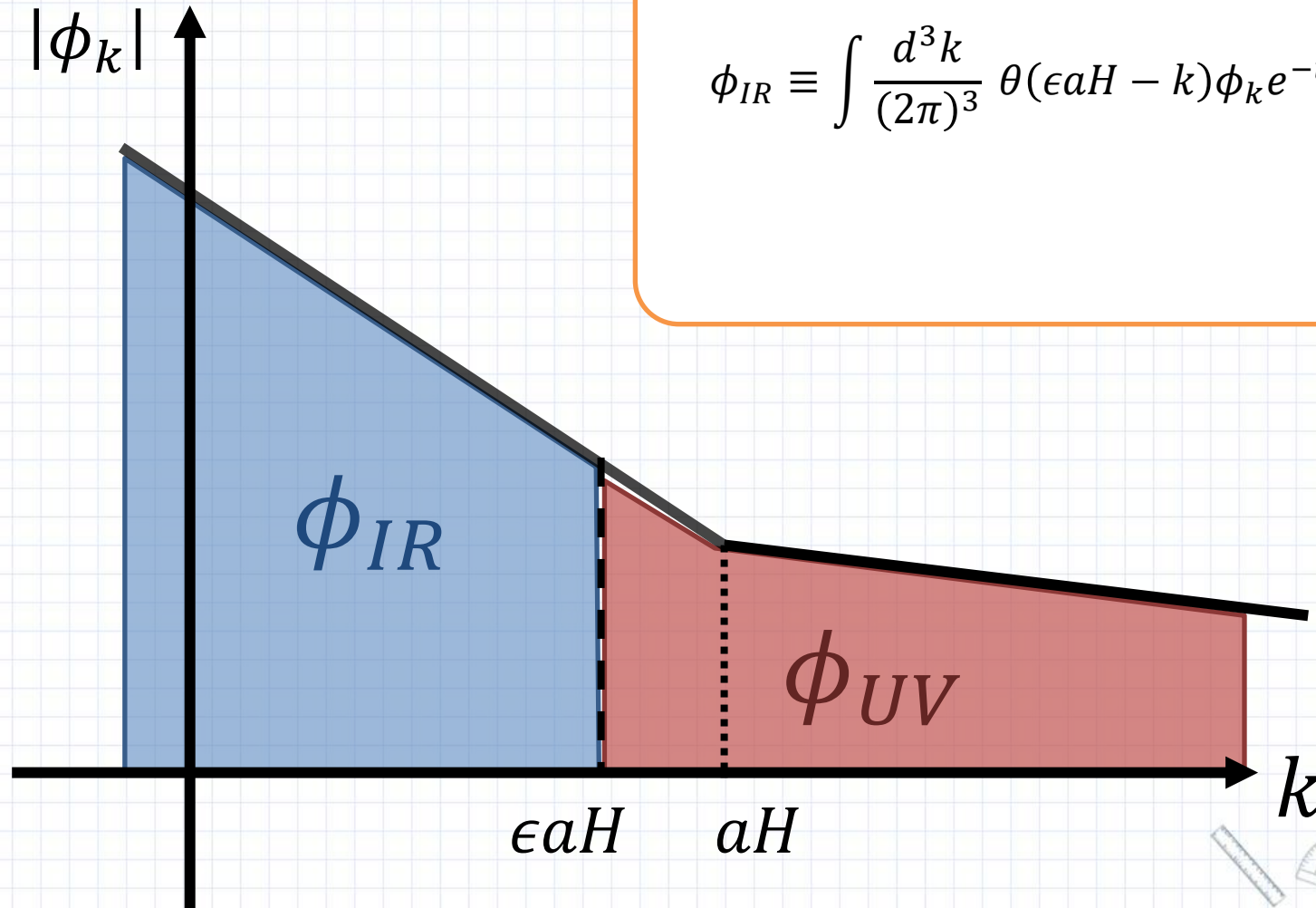


**Thank you !**

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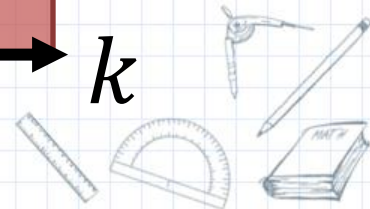


### Original S.F. for $\phi$



$$\phi(t, x) = \phi_{IR}(t, x) + \phi_{UV}(t, x)$$

$$\phi_{IR} \equiv \int \frac{d^3 k}{(2\pi)^3} \theta(\epsilon a H - k) \phi_k e^{-i k \cdot x}$$

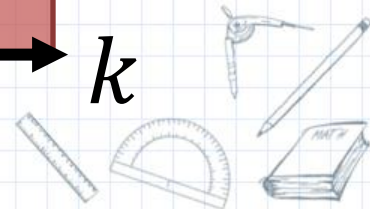
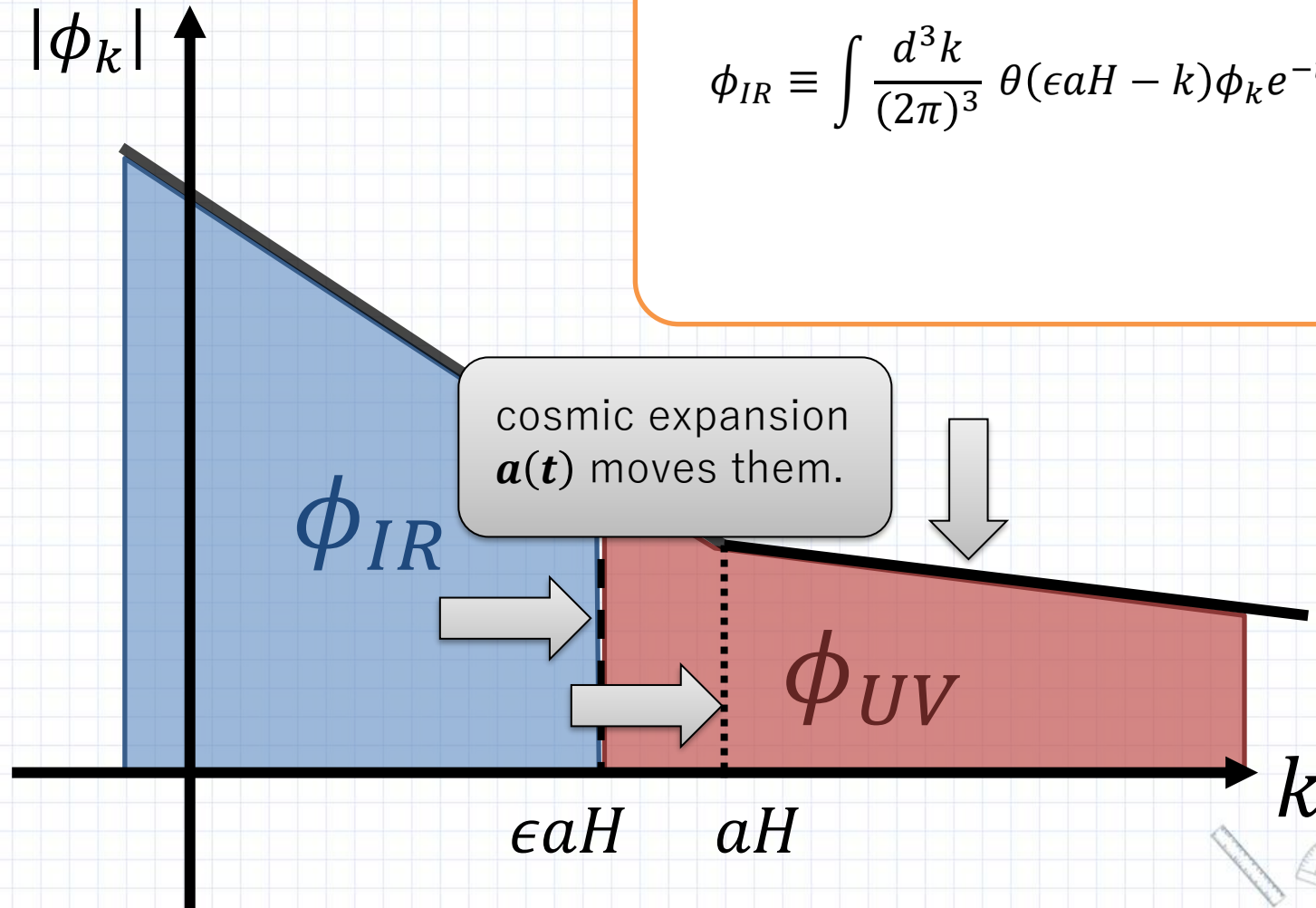




### Original S.F. for $\phi$

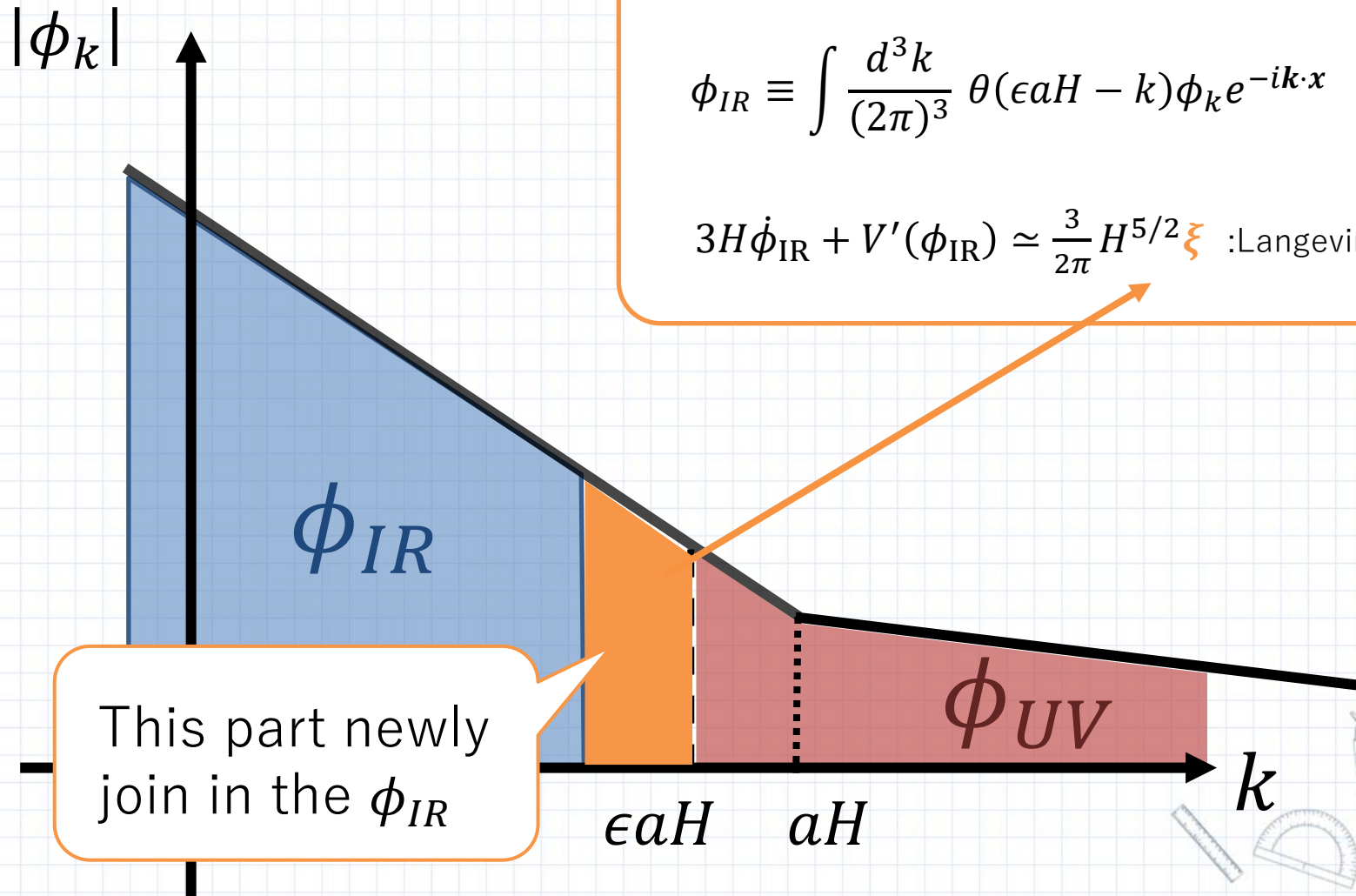
$$\phi(t, x) = \phi_{IR}(t, x) + \phi_{UV}(t, x)$$

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### Original S.F. for $\phi$

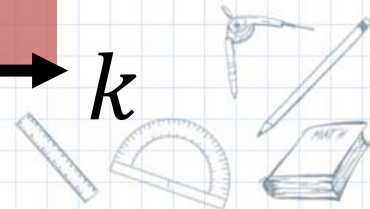


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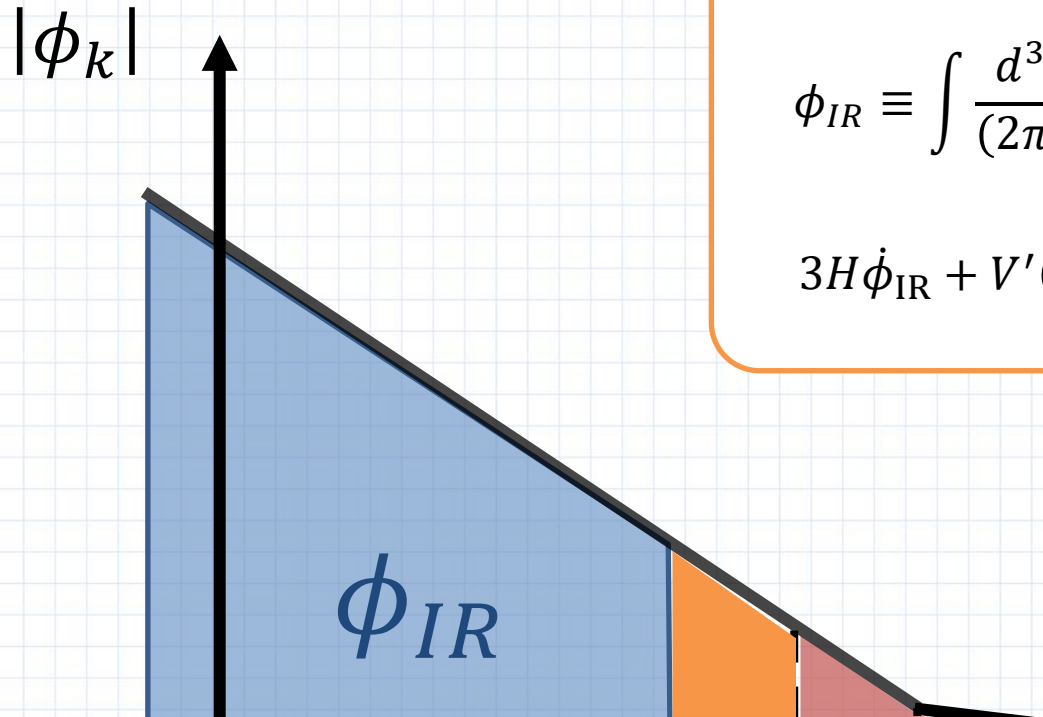
$$3H\dot{\phi}_{IR} + V'(\phi_{IR}) \simeq \frac{3}{2\pi} H^{5/2} \xi \quad \text{:Langevin eq.}$$

This part newly  
join in the  $\phi_{IR}$





### Original S.F. for $\phi$

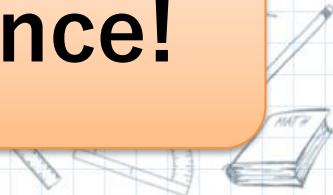


$$\phi(t, x) = \phi_{IR}(t, x) + \phi_{UV}(t, x)$$

$$\phi_{IR} \equiv \int \frac{d^3k}{(2\pi)^3} \theta(\epsilon a H - k) \phi_k e^{-ik \cdot x}$$

$$3H\dot{\phi}_{IR} + V'(\phi_{IR}) \simeq \frac{3}{2\pi} H^{5/2} \xi \quad \text{:Langevin eq.}$$

**BG + fluctuations are solved at once!**





## S.F. for EM fields (without charged particles)

The Langevin eqs. are given by

$$\begin{pmatrix} \partial_N \tilde{\mathbf{B}}_{\text{IR}} + 2\tilde{\mathbf{B}}_{\text{IR}} \\ \partial_N \tilde{\mathbf{E}}_{\text{IR}} + 2\tilde{\mathbf{E}}_{\text{IR}} + 2\xi \tilde{\mathbf{B}}_{\text{IR}} \end{pmatrix} = R^T \begin{pmatrix} 0 \\ \tilde{\xi}(N) \end{pmatrix},$$

Only **one noise term** appears, because EMFs consist of **one DoF**.

where

$$\tilde{\mathbf{E}}_{\text{IR}}^{\pm} = -a^{-2} \int \frac{d^3k}{(2\pi)^3} e^{ik \cdot x} \theta(\kappa a H - k) \boldsymbol{\epsilon}^{\pm}(\hat{\mathbf{k}}) \partial_{\tau} \mathcal{A}_{\pm}(\tau, k) \hat{a}_{\mathbf{k}}^{\pm} + h.c.$$

$$\tilde{\mathbf{B}}_{\text{IR}}^{\pm} = a^{-2} \nabla \times \int \frac{d^3k}{(2\pi)^3} e^{ik \cdot x} \theta(\kappa a H - k) \boldsymbol{\epsilon}^{\pm}(\hat{\mathbf{k}}) \mathcal{A}_{\pm}(\tau, k) \hat{a}_{\mathbf{k}}^{\pm} + h.c.$$

and  $R, \xi$  are fixed by the mode function  $\mathcal{A}_{+}(\tau, k)$  at  $k = \kappa a H$

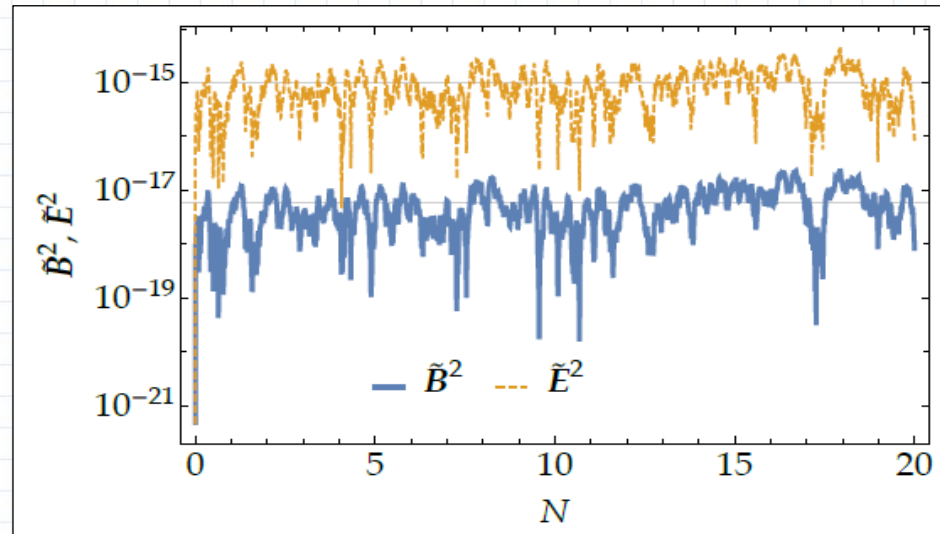
We can solve it once  $\mathcal{A}_{+}(\tau, k)$  is obtained.

$$R = \frac{1}{\sqrt{|W|^2 + |W'|^2}} \begin{pmatrix} |W'| & |W| \\ -|W| & |W'| \end{pmatrix},$$

$$\langle \tilde{\xi}_i(N) \tilde{\xi}_j(N') \rangle = \delta_{ij} \delta(N - N') \frac{\kappa^4 H^4}{12\pi^2} e^{\pi\xi} (|W|^2 + |W'|^2),$$



## Stochastic Results 1



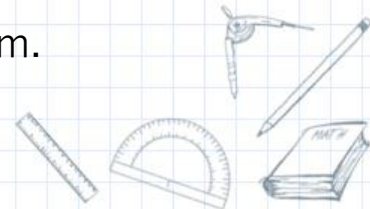
We can compute

$$\langle \tilde{\mathbf{B}}_{\text{IR}}^2(t) \rangle \simeq \frac{1}{4} \tilde{\mathcal{P}}_{BB}(\kappa),$$

the variance of  $\tilde{\mathbf{E}}_{\text{IR}}, \tilde{\mathbf{B}}_{\text{IR}}$

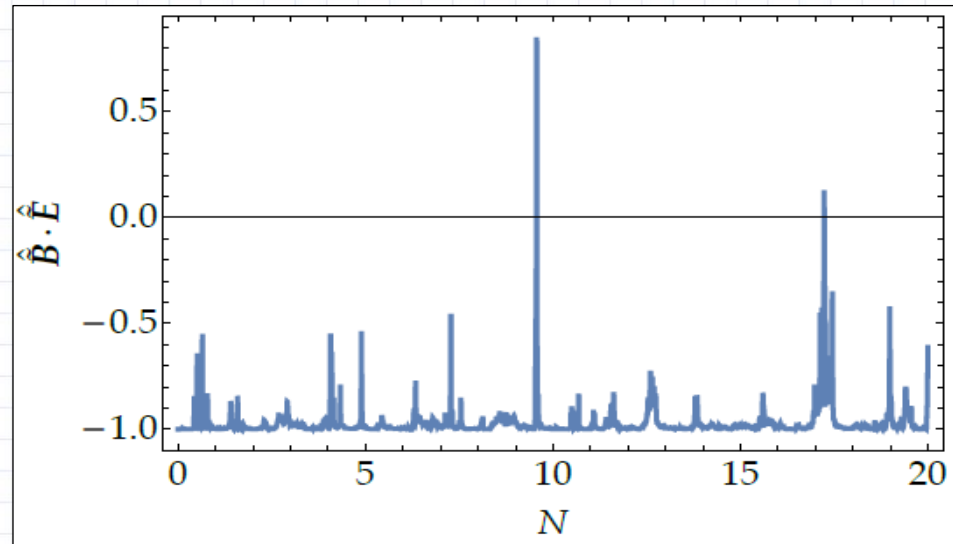
$$\langle \tilde{\mathbf{E}}_{\text{IR}}^2(t) \rangle \simeq \frac{1}{4} \tilde{\mathcal{P}}_{EE}(\kappa) - \frac{\xi}{8} (\tilde{\mathcal{P}}_{BE}(\kappa) + \tilde{\mathcal{P}}_{EB}(\kappa)) + \frac{\xi^2}{8} \tilde{\mathcal{P}}_{BB}(\kappa).$$

Indeed, in a realization, the  $\tilde{\mathbf{E}}_{\text{IR}}, \tilde{\mathbf{B}}_{\text{IR}}$  strengths fluctuate around them. Clearly,  $\tilde{\mathbf{E}}_{\text{IR}}$  is much stronger than  $\tilde{\mathbf{B}}_{\text{IR}}$ .





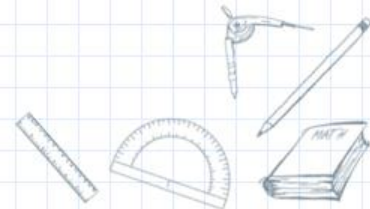
## Stochastic Results 2



$\tilde{\mathbf{E}}_{\text{IR}}$  &  $\tilde{\mathbf{B}}_{\text{IR}}$  are **anti-parallel**, which can be analytically confirmed:

$$\hat{\mathbf{E}}_{\text{IR}} \cdot \hat{\mathbf{B}}_{\text{IR}} = \frac{\langle \tilde{\mathbf{E}}_{\text{IR}} \cdot \tilde{\mathbf{B}}_{\text{IR}} + \tilde{\mathbf{B}}_{\text{IR}} \cdot \tilde{\mathbf{E}}_{\text{IR}} \rangle}{2 \langle \tilde{\mathbf{B}}_{\text{IR}}^2 \rangle^{1/2} \langle \tilde{\mathbf{E}}_{\text{IR}}^2 \rangle^{1/2}} \xrightarrow{\kappa \ll 2\xi} - \frac{|W'| + \xi|W|/2}{\sqrt{|W'|^2 + \xi|W'||W| + \xi^2|W|^2/2}} \xrightarrow{2\kappa\xi \ll 1} -1,$$

This is because the coupling  $\phi F \tilde{F} = \phi \mathbf{E} \cdot \mathbf{B}$  sources not  $\mathbf{E} \perp \mathbf{B}$  but  $\mathbf{E} \parallel \mathbf{B}$ .







### Stochastic Results 3

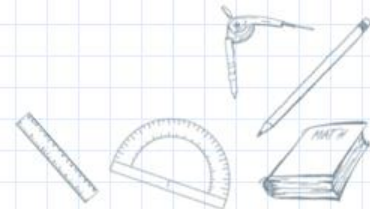
The coherent time of  $\tilde{\mathbf{E}}_{\text{IR}}, \tilde{\mathbf{B}}_{\text{IR}}$  is

$$\tau_c = (2H)^{-1}$$

$$\langle \tilde{\mathbf{B}}_{\text{IR}}(t) \cdot \tilde{\mathbf{B}}_{\text{IR}}(t + \Delta t) \rangle \simeq e^{-2H\Delta t} \tilde{\mathbf{B}}_{\text{IR}}^2(t),$$

$$\langle \tilde{\mathbf{E}}_{\text{IR}}(t) \cdot \tilde{\mathbf{E}}_{\text{IR}}(t + \Delta t) \rangle \simeq e^{-2H\Delta t} [\tilde{\mathbf{E}}_{\text{IR}}^2(t) - 2\xi H\Delta t \tilde{\mathbf{E}}_{\text{IR}}(t) \cdot \tilde{\mathbf{B}}_{\text{IR}}(t)].$$

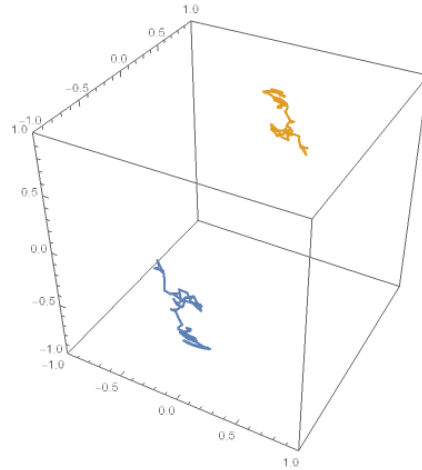
In other words,  $\tilde{\mathbf{E}}_{\text{IR}}, \tilde{\mathbf{B}}_{\text{IR}}$  lose their memories every 0.5 e-folds and they are always dominated by newly produced fluctuations.



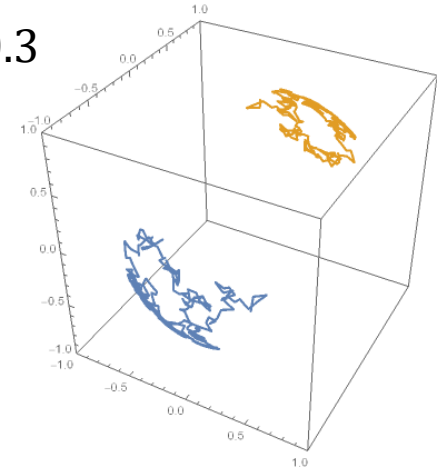
### Stochastic Results 3

$\tilde{\mathbf{E}}_{\text{IR}}$  &  $\tilde{\mathbf{B}}_{\text{IR}}$  are anti-parallel and randomly rotate for  $\Delta N = 0.5$

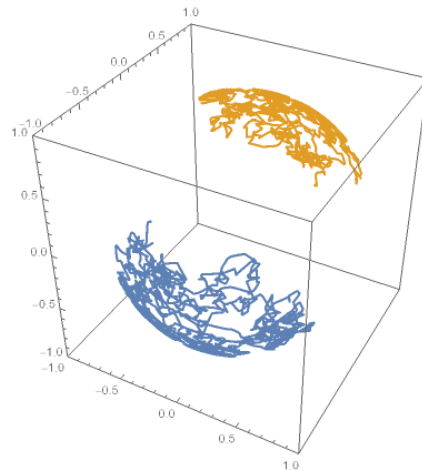
①  $N = 0.1$



②  $N = 0.3$



③  $N = 1$



④  $N = 3$

