Reconnection-controlled decay of primordial magnetic fields

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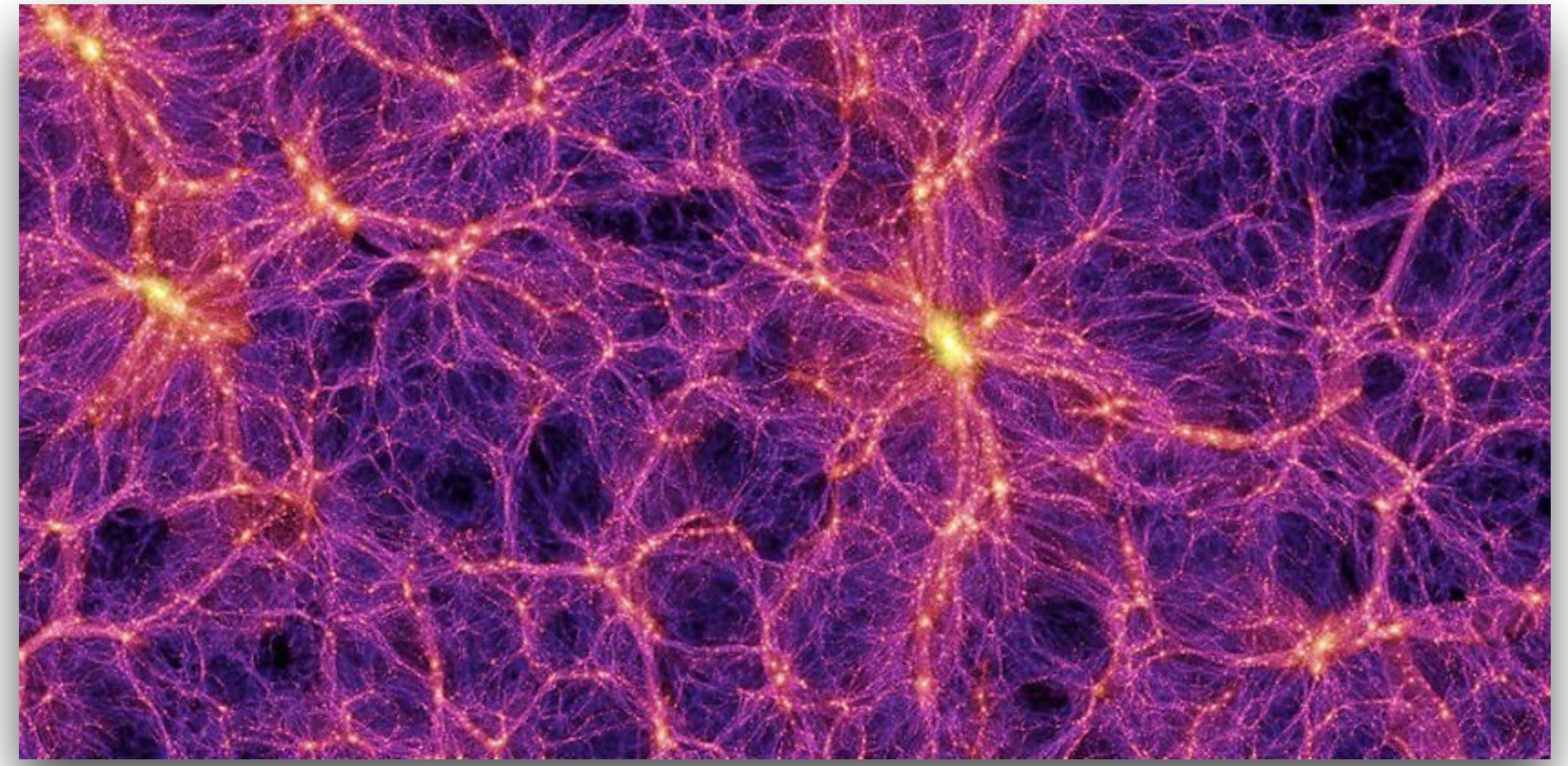
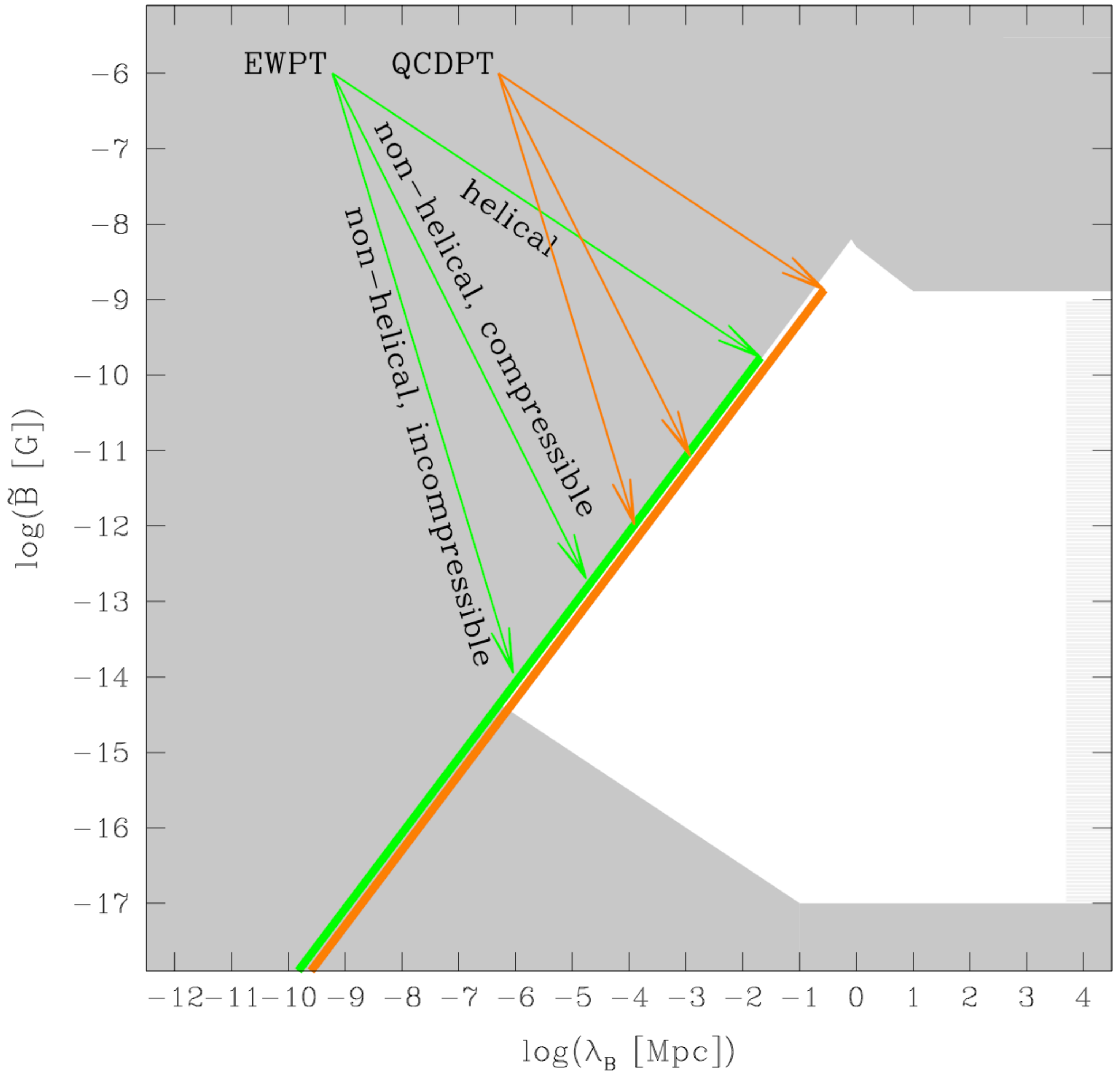
The problem

Primordial magnetic fields might have been generated during phase transitions or inflation in the early Universe. How strong would the relics of those fields be today?



Millenium Simulation, Springel et. al. 2005



Durrer & Neronov *2013*

Evolution of primordial magnetic fields

Primordial magnetic fields (PMFs) were born into a hot plasma composed of collisionally coupled quarks/protons, electrons, photons and neutrinos. We can use a fluid theory to describe their evolution (Brandenburg+ 1996):

$$\mathrm{d}s^2 = a^2(t)(-\mathrm{d}t^2 + \mathrm{d}x_i\,\mathrm{d}x^i)$$

$$\tilde{\rho} = a^4\rho, \quad \tilde{p} = a^4 p, \quad \tilde{\mathbf{B}} = a^2\mathbf{B}, \quad \tilde{\mathbf{u}} = \mathbf{u},$$
$$\tilde{\eta} = \eta/a, \quad \tilde{\nu} = \nu/a,$$

$$\frac{\partial \tilde{\mathbf{u}}}{\partial t} + \tilde{\mathbf{u}}\cdot\nabla\tilde{\mathbf{u}} = -\nabla\tilde{p} + (\nabla\times\tilde{\mathbf{B}})\times\tilde{\mathbf{B}} + \tilde{\nu}\nabla^2\tilde{\mathbf{u}},$$

$$\frac{\partial \tilde{\mathbf{B}}}{\partial t} = \nabla\times(\tilde{\mathbf{u}}\times\tilde{\mathbf{B}}) + \tilde{\eta}\nabla^2\tilde{\mathbf{B}},$$

$$\nabla\cdot\tilde{\mathbf{u}} = 0.$$

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$$\beta \equiv \frac{2\tilde{p}}{\tilde{B}^2} \gg 1 \qquad \mathrm{Pm} \equiv \frac{\tilde{\nu}}{\tilde{\eta}} \gg 1$$

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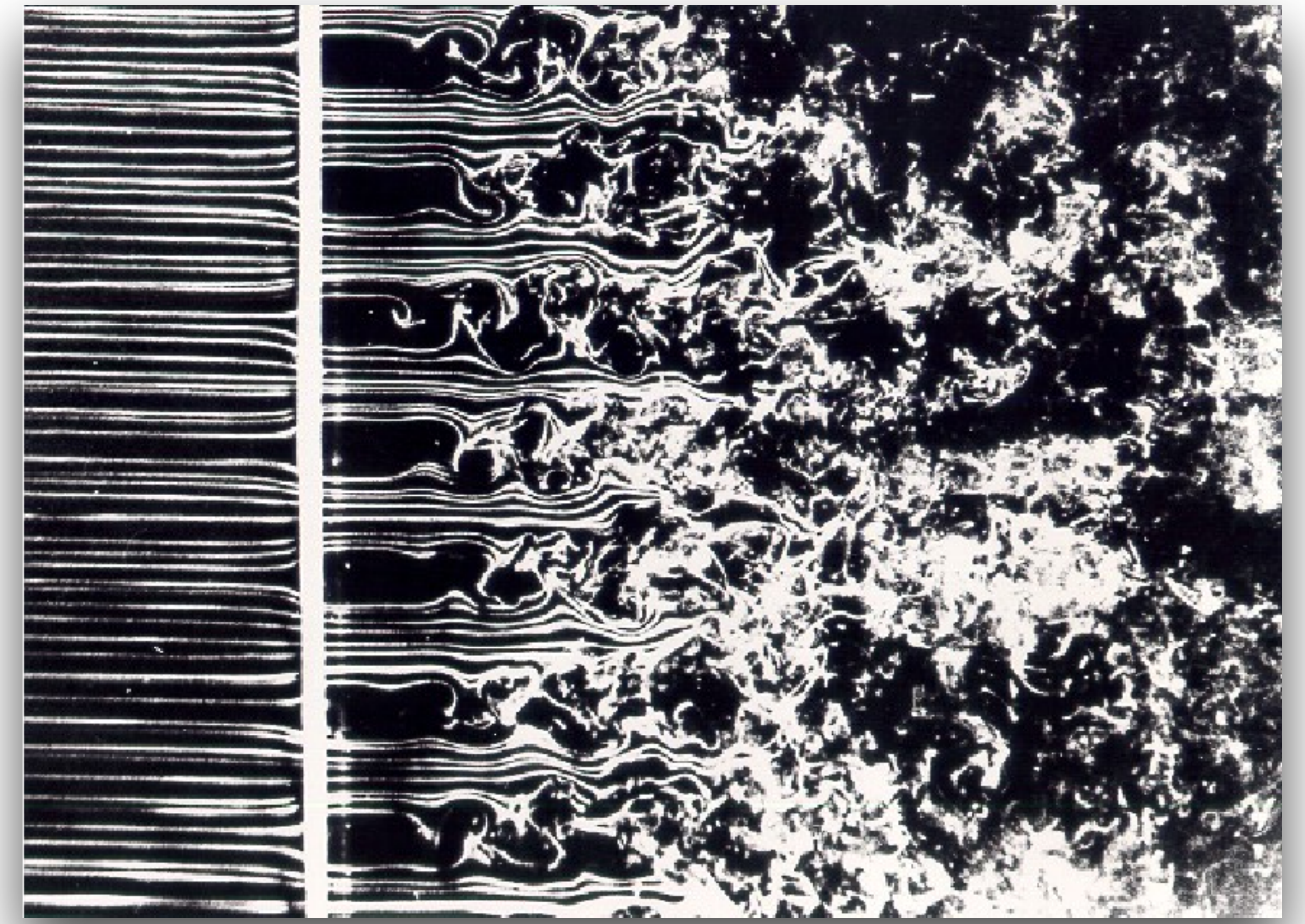
PMFs would have experienced turbulent MHD decay between magnetogenesis and recombination.

Decaying turbulence: Kolmogorov's philosophy

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\frac{E}{\tau(E, \lambda, \dots)} \implies E \propto t^{-p}$$

$$I = I(E, \lambda) = \text{constant} \implies \lambda \sim E^{\alpha}$$



Van Dyke, *Album of Fluid Motion* #152, 1982

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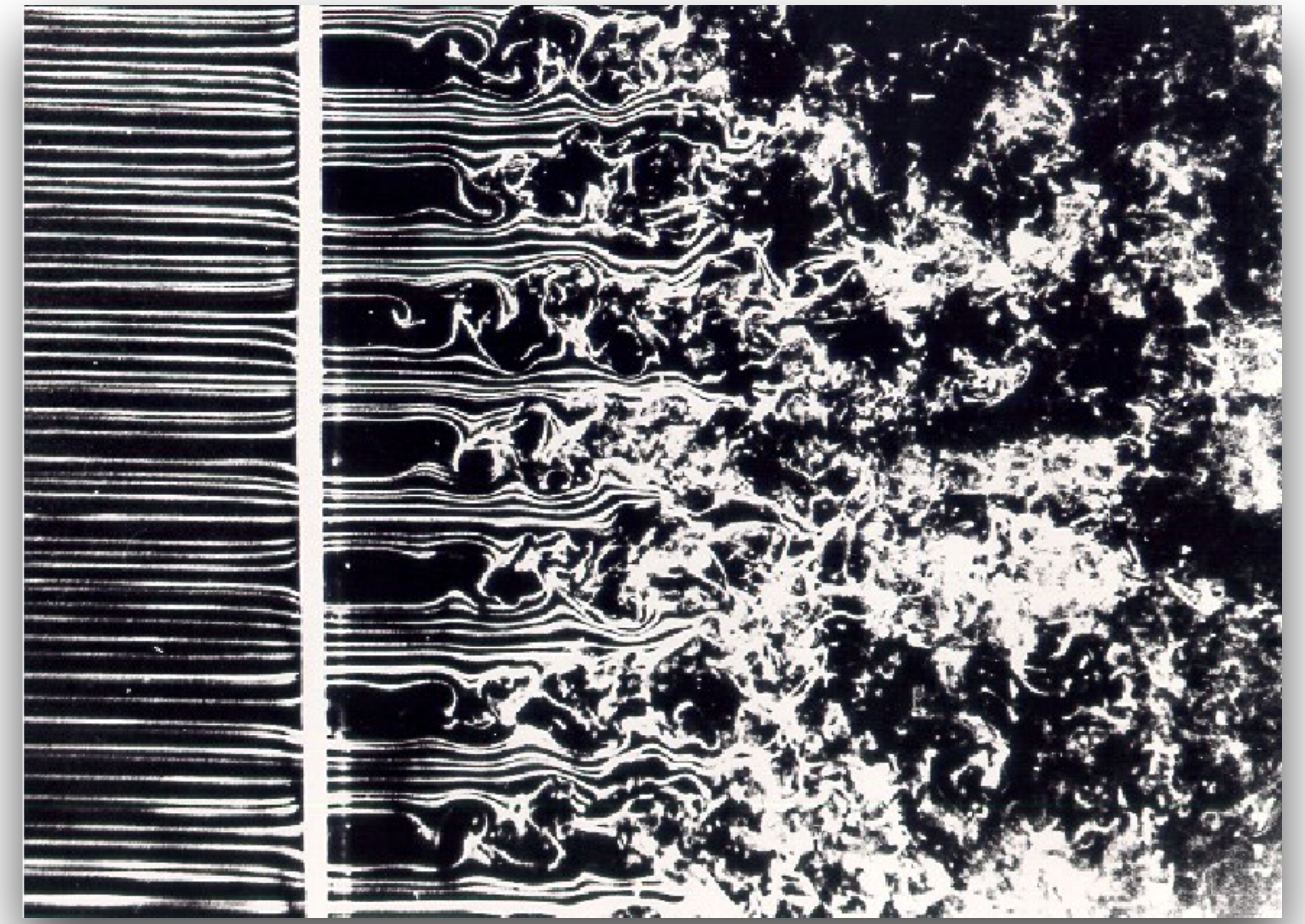
Canonical example: hydrodynamic turbulence

$$I = -\int \mathrm{d}^3\boldsymbol{r}\, r^2 \langle \boldsymbol{u}(\boldsymbol{x}) \cdot \boldsymbol{u}(\boldsymbol{x}+\boldsymbol{r}) \rangle \implies U^2 L^5 \sim E L^5 \sim \text{const}$$

$$\tau \sim \frac{\lambda}{U} \implies \frac{\mathrm{d}E}{\mathrm{d}t} \sim -\frac{E^{3/2}}{\lambda} \sim E^{17/10} \implies E \propto t^{-10/7},\ \lambda \propto t^{2/7}$$

Kolmogorov 1943



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The “largest-processed-eddy relation”

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If $\tau \sim E^{\alpha} t^{\beta}$, then

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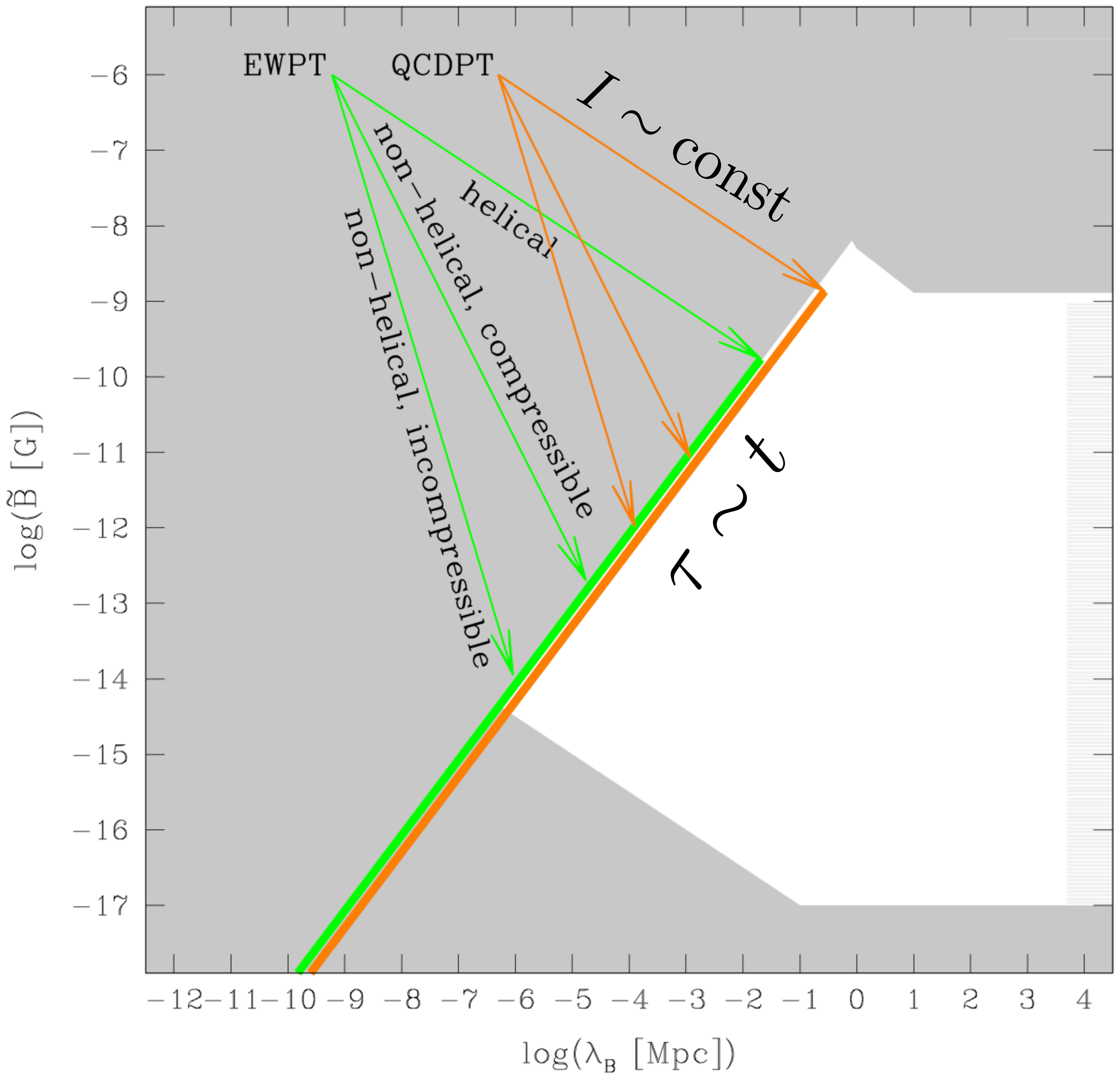

Adapted from Durrer & Neronov 2013

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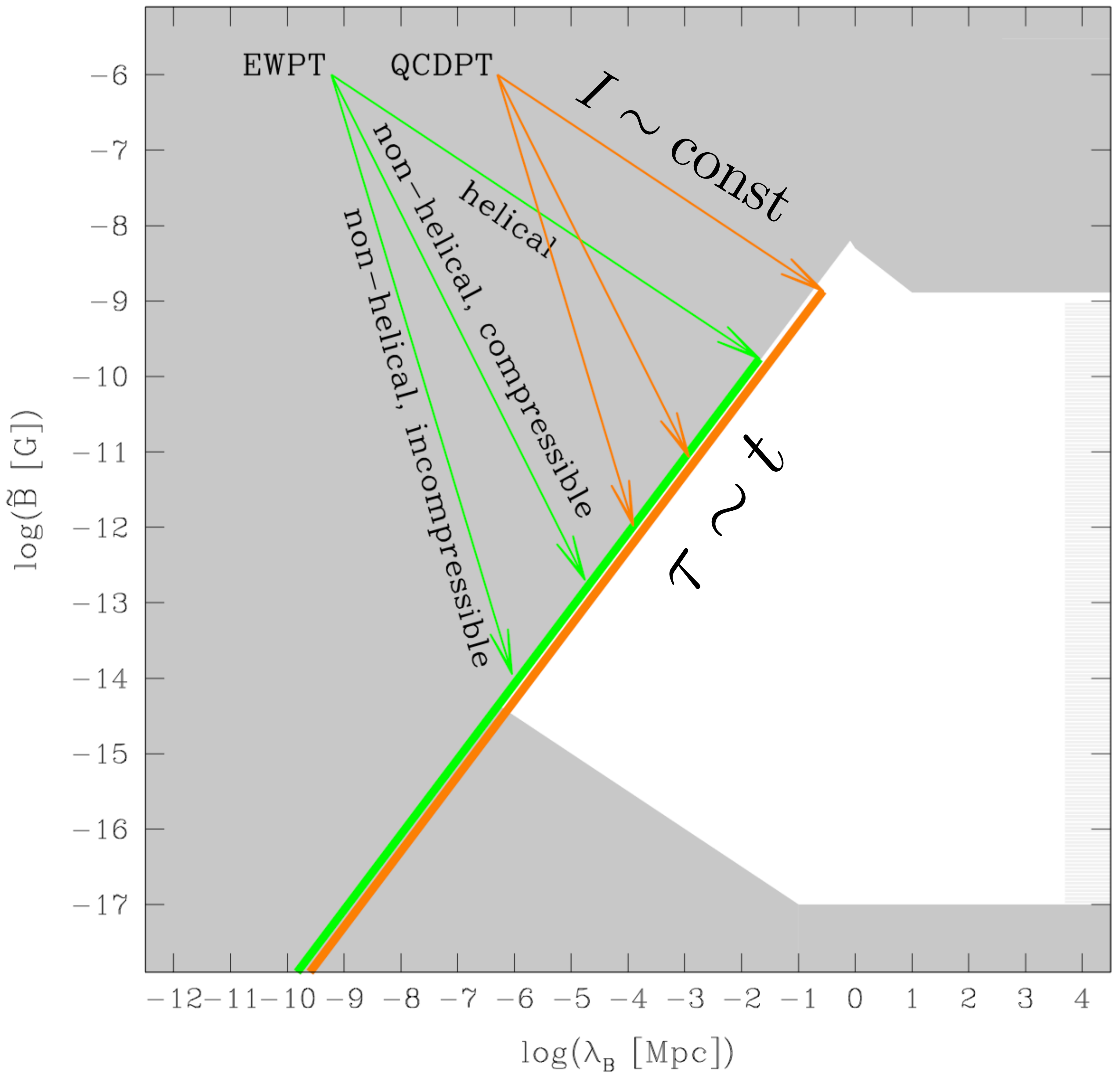

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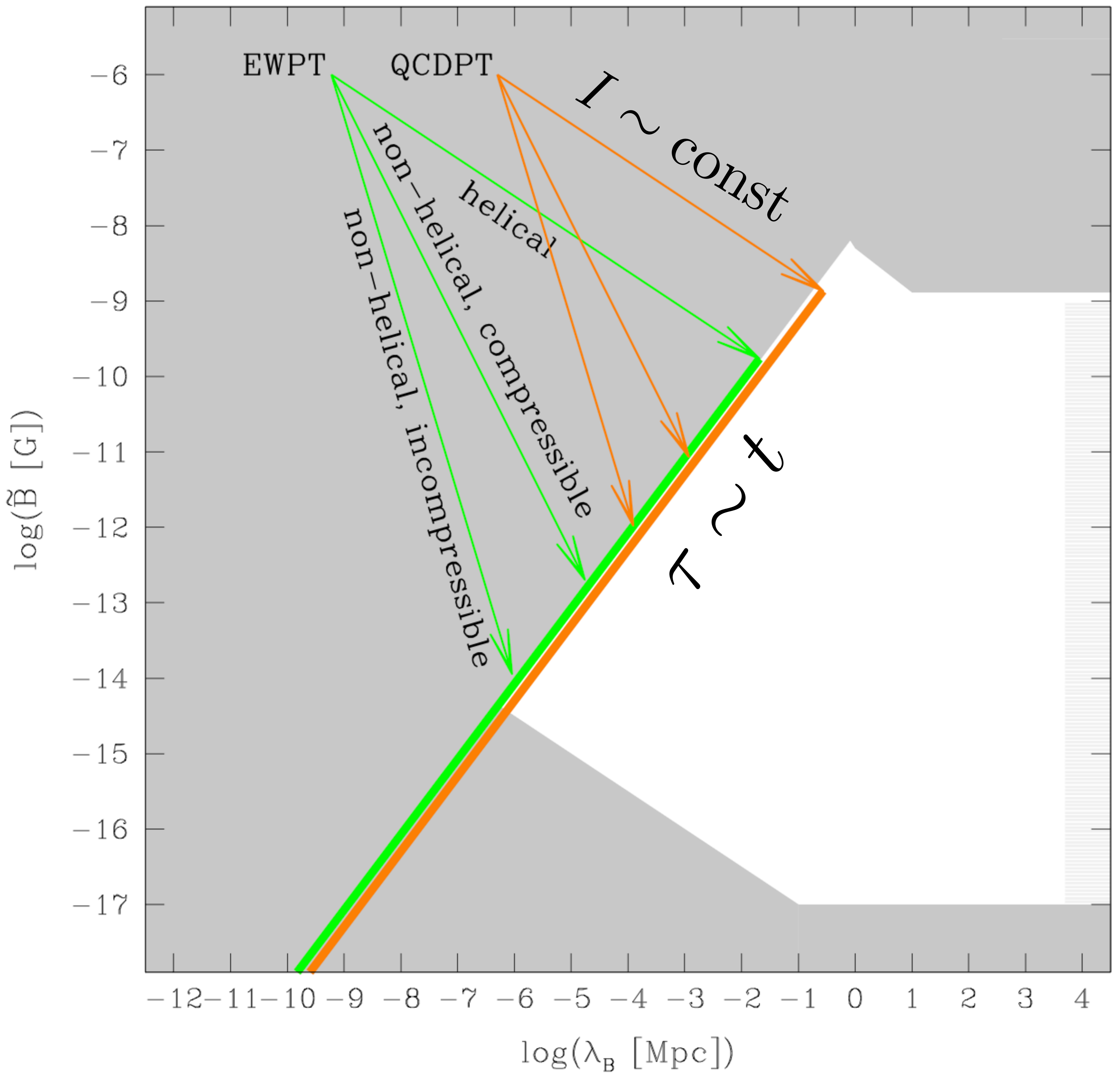

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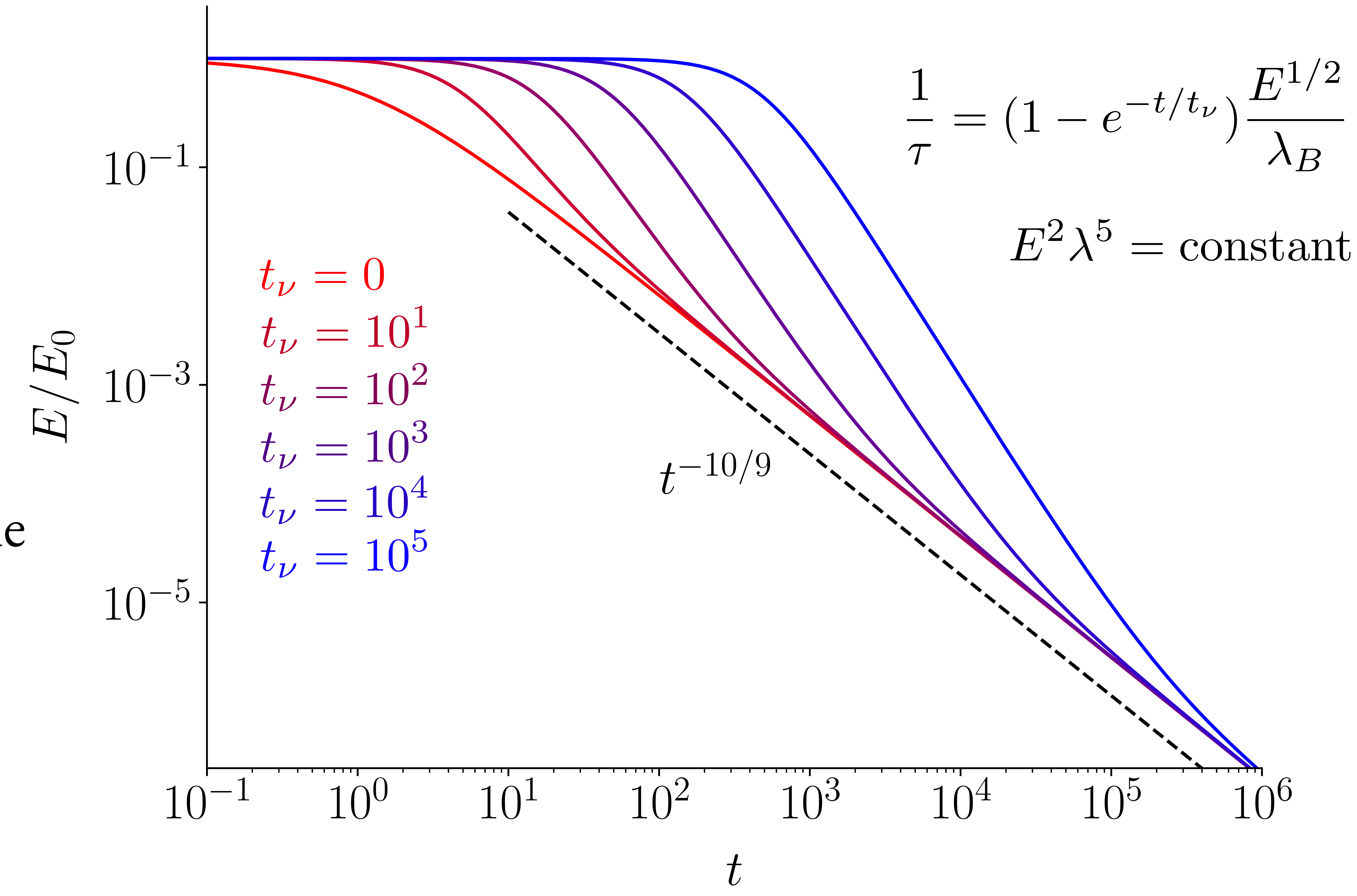
Simple model of neutrino decoupling:



The problem of predicting the evolution of PMFs has two parts:

1. What is $I(E, \lambda)$?
2. What is $\tau(E, \lambda)$ at recombination?

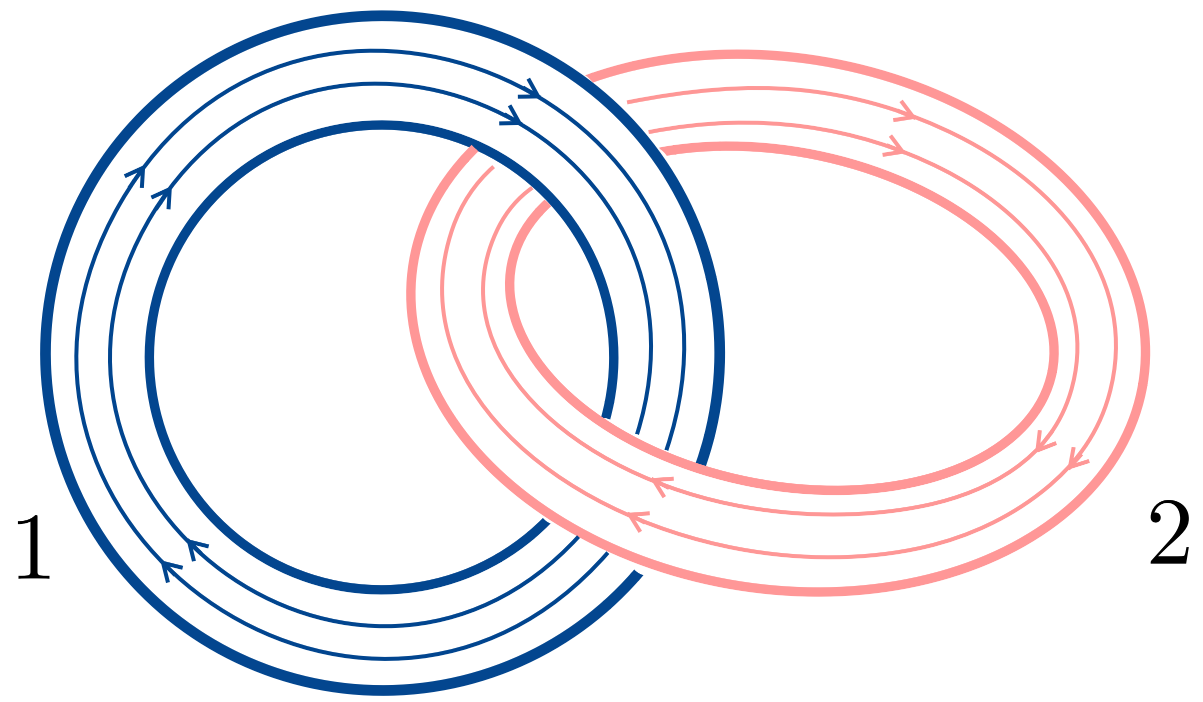
These questions are distinct in principle, but I argue that the answer to the latter is informed by the former.

The role of magnetic topology: JB Taylor relaxation

Consider a magnetic configuration consisting of N closed flux tubes. For each one, the helicity

$$H_i = \int_{V_i} \mathrm{d}^3\boldsymbol{x}\, \boldsymbol{A} \cdot \boldsymbol{B}$$

under ideal (flux-frozen) dynamics. We know from Moffatt (1978) that this quantity has a topological interpretation: it is the flux linked by tube i.



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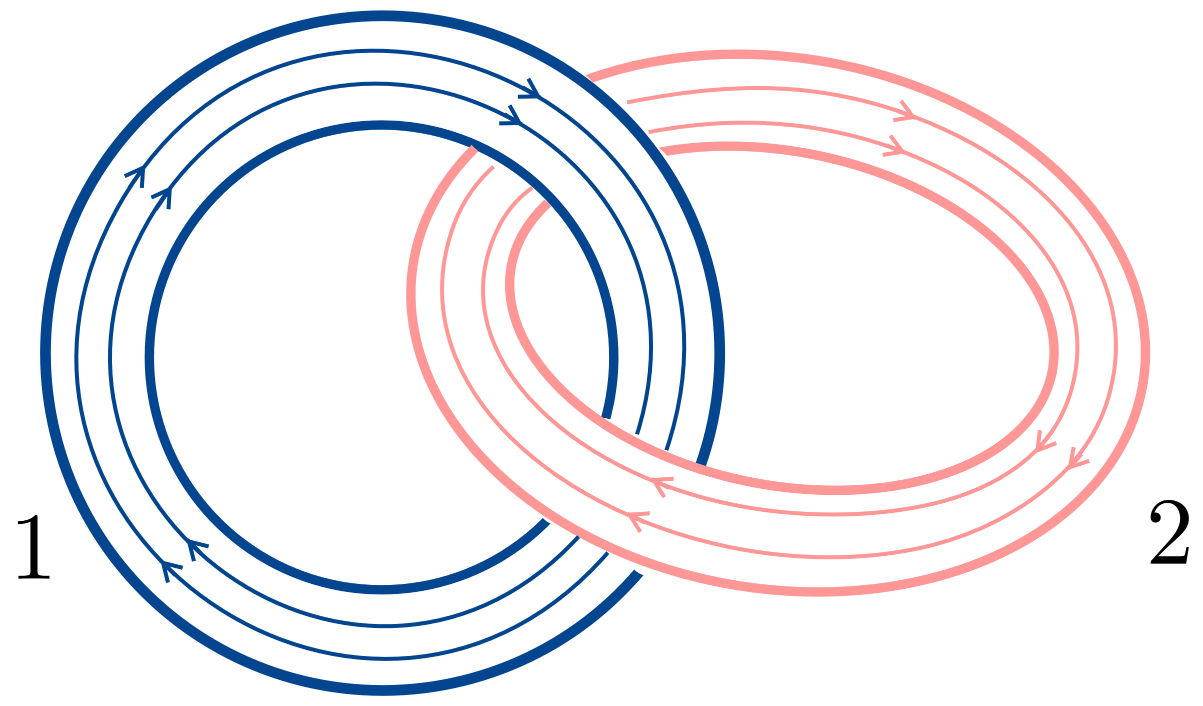

$$H_1 = \Phi_1 \Phi_2$$

Thus, under topology-preserving relaxation, we must have that $\boldsymbol{B}_{\text{final}} = \boldsymbol{B}_{\text{final}}(H_1, H_2, \ldots, H_N)$.

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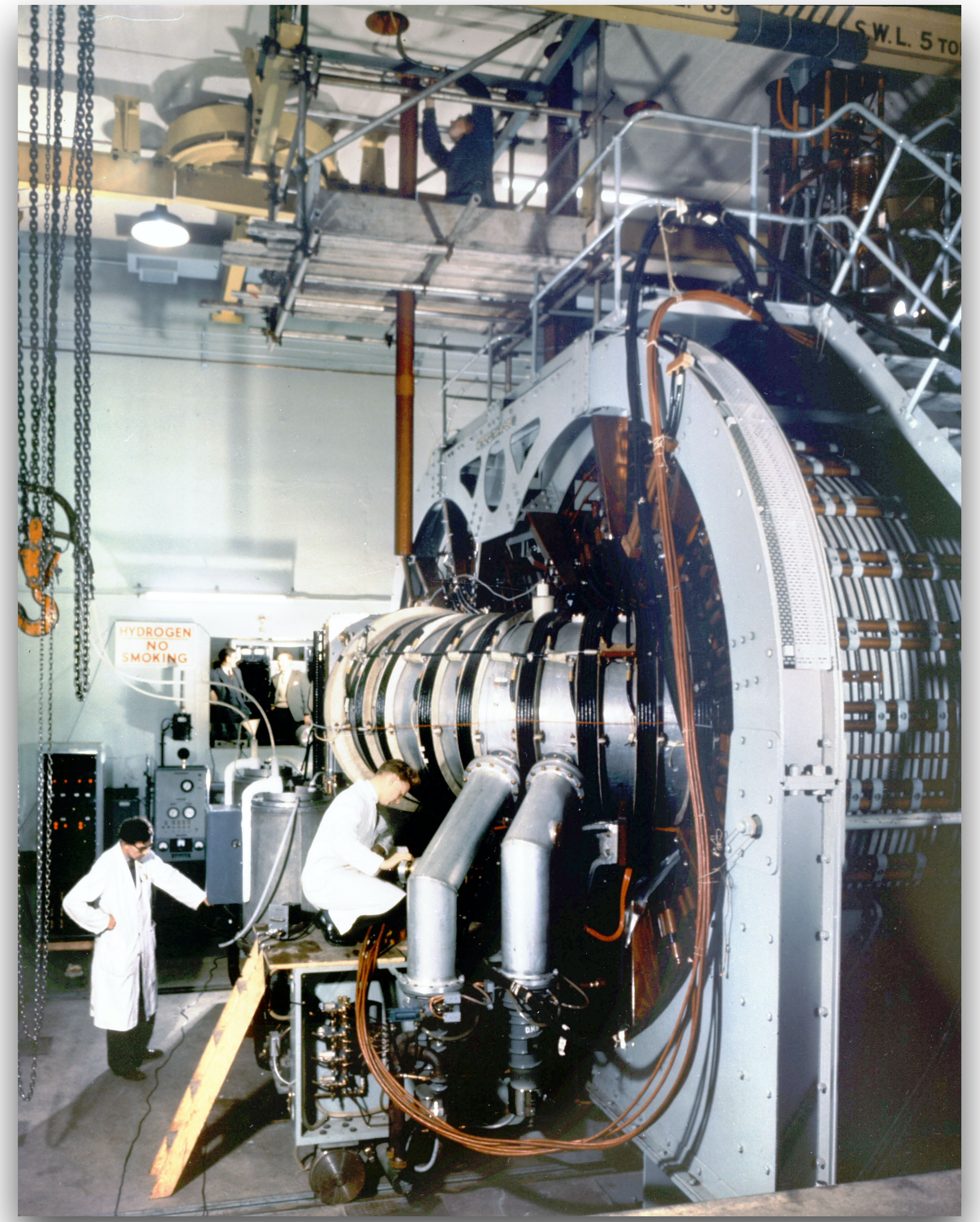
This is inconsistent with experiment (both numerical and real-life): the final state of relaxation is only weakly dependent on initial conditions.

> ... in the quiescent phase the plasma profile is almost independent of any details of the initial state and depends principally on the 'pinch ratio' $\theta = B_\theta / B_0$, where B_θ is the poloidal field at the plasma boundary.

J. B. Taylor & S. L. Newton (2015)



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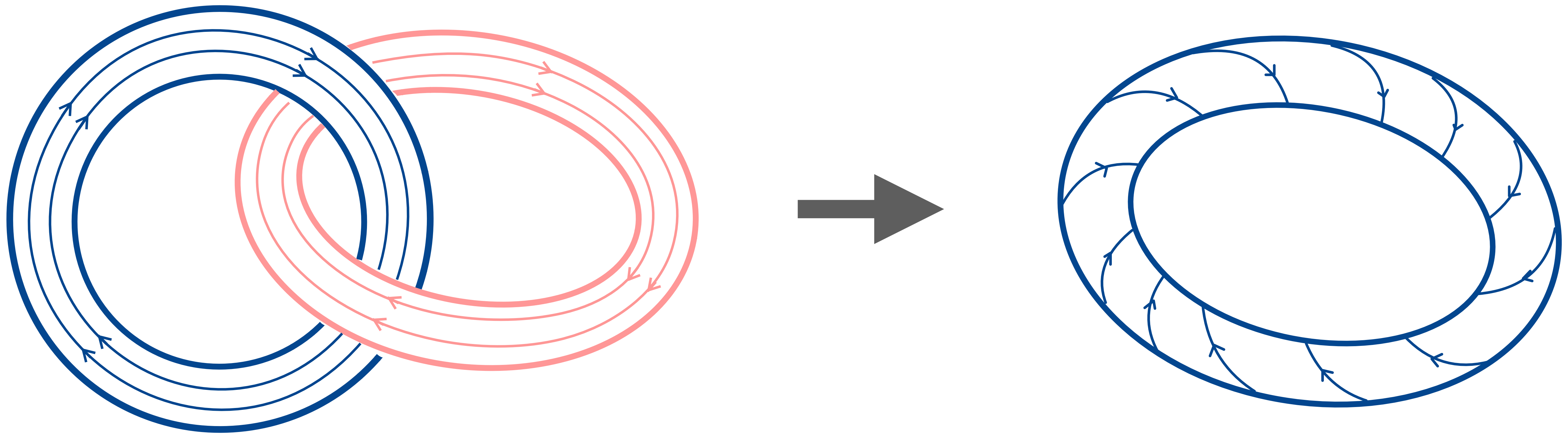
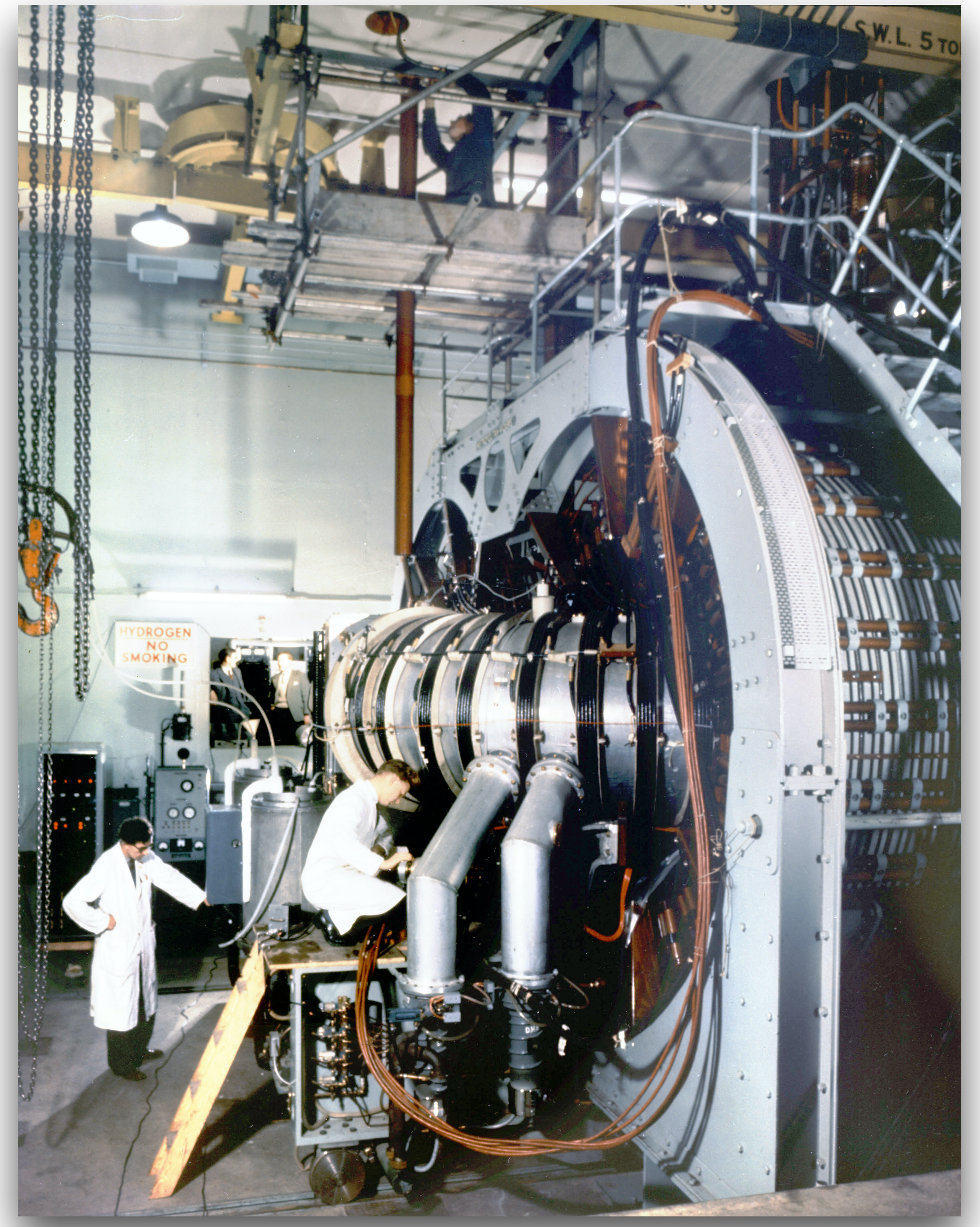
It is not difficult to see why — in real plasma, field lines are not perfectly frozen in. With any finite resistivity, they can *reconnect* and thus access new, lower-energy states.





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Remarkably,

$$H = \sum_i H_i = \int \mathrm{d}^3\boldsymbol{x}\,\boldsymbol{A}\cdot\boldsymbol{B}$$

remains constant even under reconnection of magnetic-field lines.

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Why?

$$\frac{\partial h}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{F} = -2\eta \boldsymbol{B} \cdot (\boldsymbol{\nabla} \times \boldsymbol{B}), \quad h = \boldsymbol{A} \cdot \boldsymbol{B}, \quad \boldsymbol{F} = \boldsymbol{u}(\boldsymbol{A} \cdot \boldsymbol{B}) - \boldsymbol{B}(\boldsymbol{A} \cdot \boldsymbol{u}) - \chi \boldsymbol{B} - \eta \boldsymbol{A} \times (\boldsymbol{\nabla} \times \boldsymbol{B})$$

Thus, if

$$\frac{\mathrm{d}E_M}{\mathrm{d}t} = -2\eta \int \mathrm{d}^3\boldsymbol{x} |\boldsymbol{\nabla} \times \boldsymbol{B}|^2 = \text{finite} \quad \text{then} \quad \frac{\mathrm{d}H}{\mathrm{d}t} = -2\eta \int \mathrm{d}^3\boldsymbol{x} \boldsymbol{B} \cdot (\boldsymbol{\nabla} \times \boldsymbol{B}) \to 0$$

$\nabla \sim \eta^{-1/2}$.

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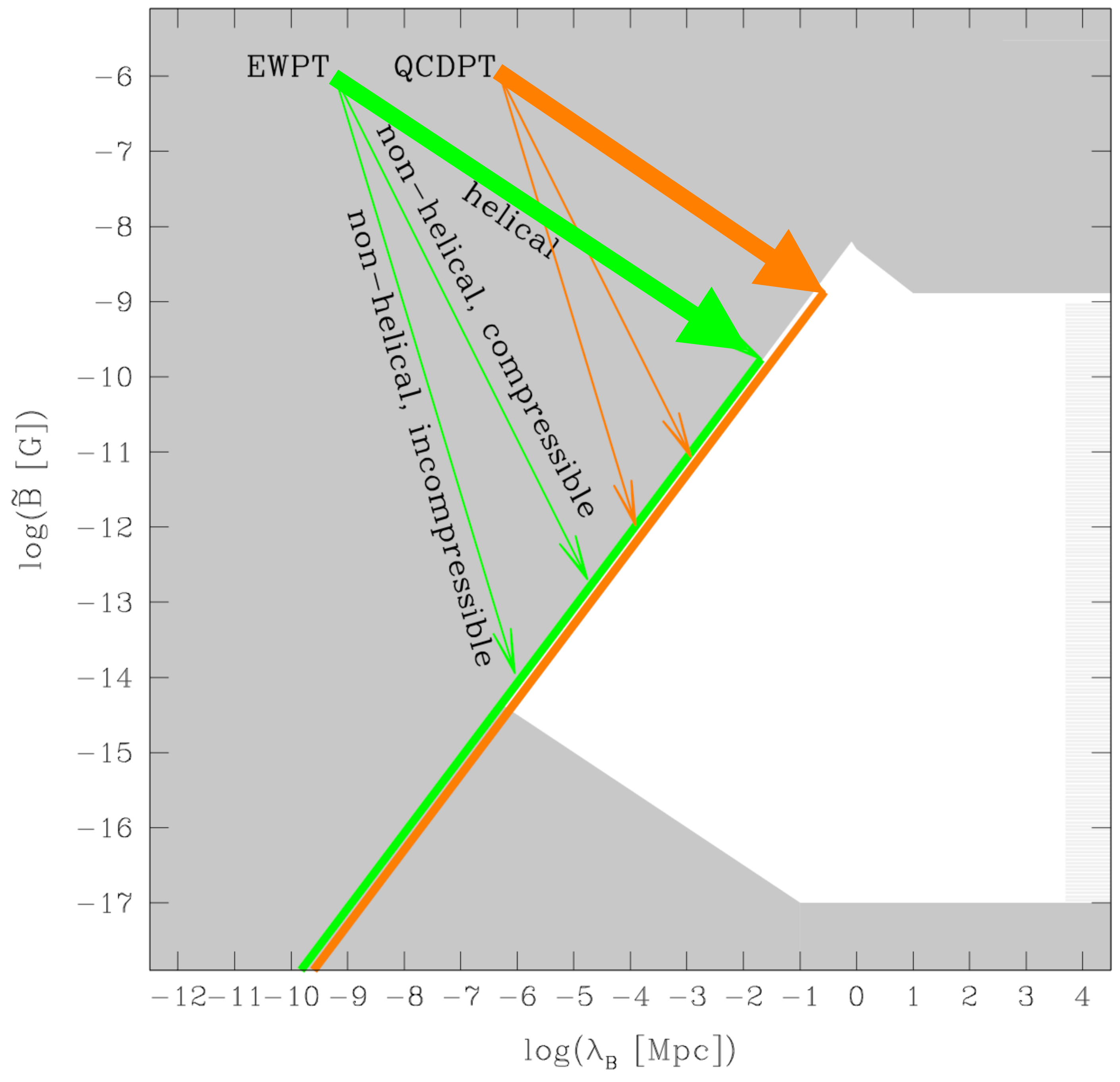
$$H \sim B^2\lambda V \sim \text{const} \implies B^2\lambda \sim \text{const}$$



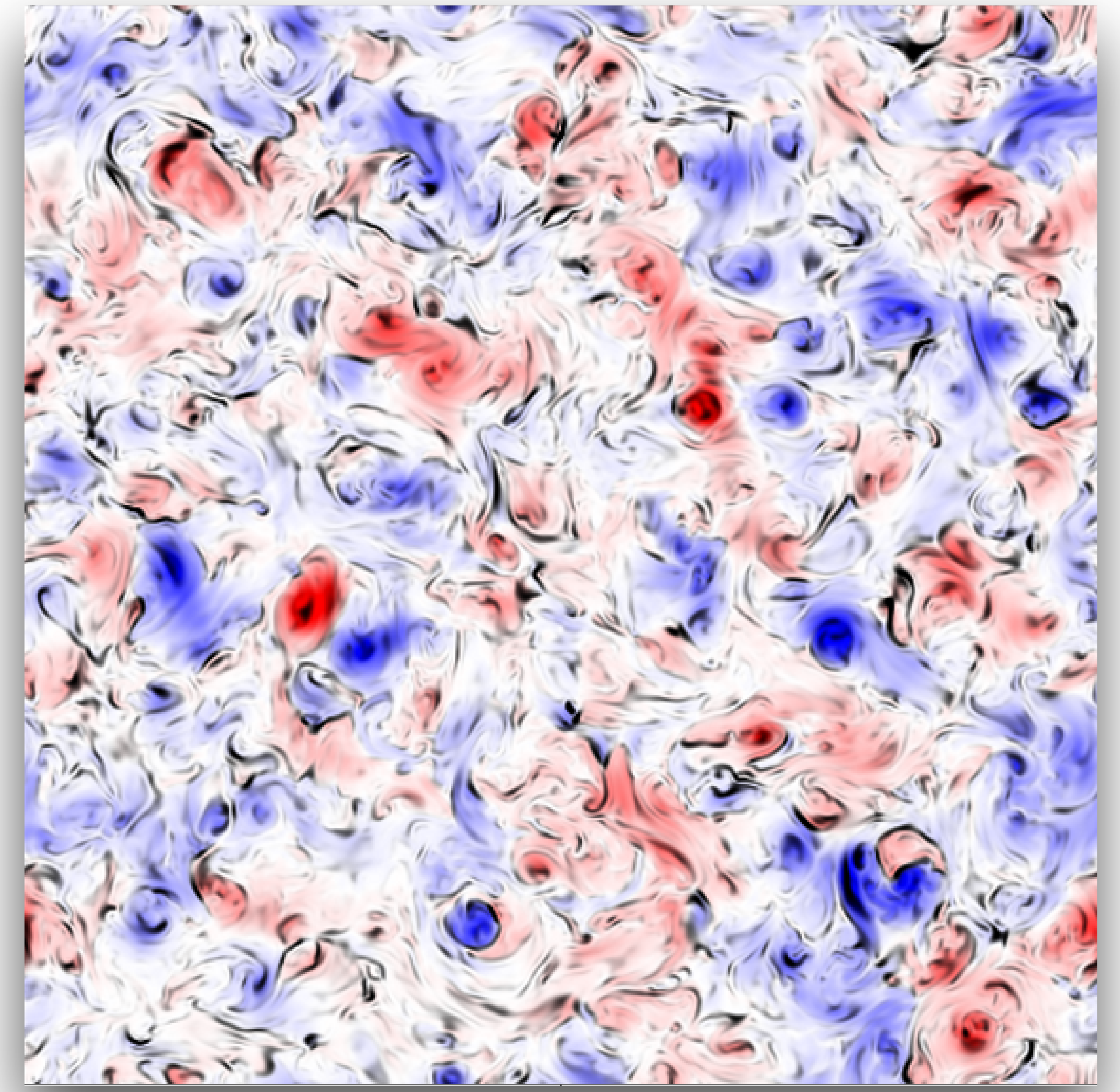
Adapted from Durrer & Neronov 2013

Helicity constrains "nonhelical" decay

The conservation law

$$\frac{\partial h}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{F} = 0, \quad h = \boldsymbol{A} \cdot \boldsymbol{B}, \quad \boldsymbol{F} = \boldsymbol{u}(\boldsymbol{A} \cdot \boldsymbol{B}) - \boldsymbol{B}(\boldsymbol{A} \cdot \boldsymbol{u}) - \chi \boldsymbol{B} - \eta \boldsymbol{A} \times (\boldsymbol{\nabla} \times \boldsymbol{B})$$

implies the existence of a second invariant in the case that $H = 0$ globally but $h \neq 0$ locally.



2D slice of $h = \boldsymbol{A} \cdot \boldsymbol{B}$ from simulation of MHD decay

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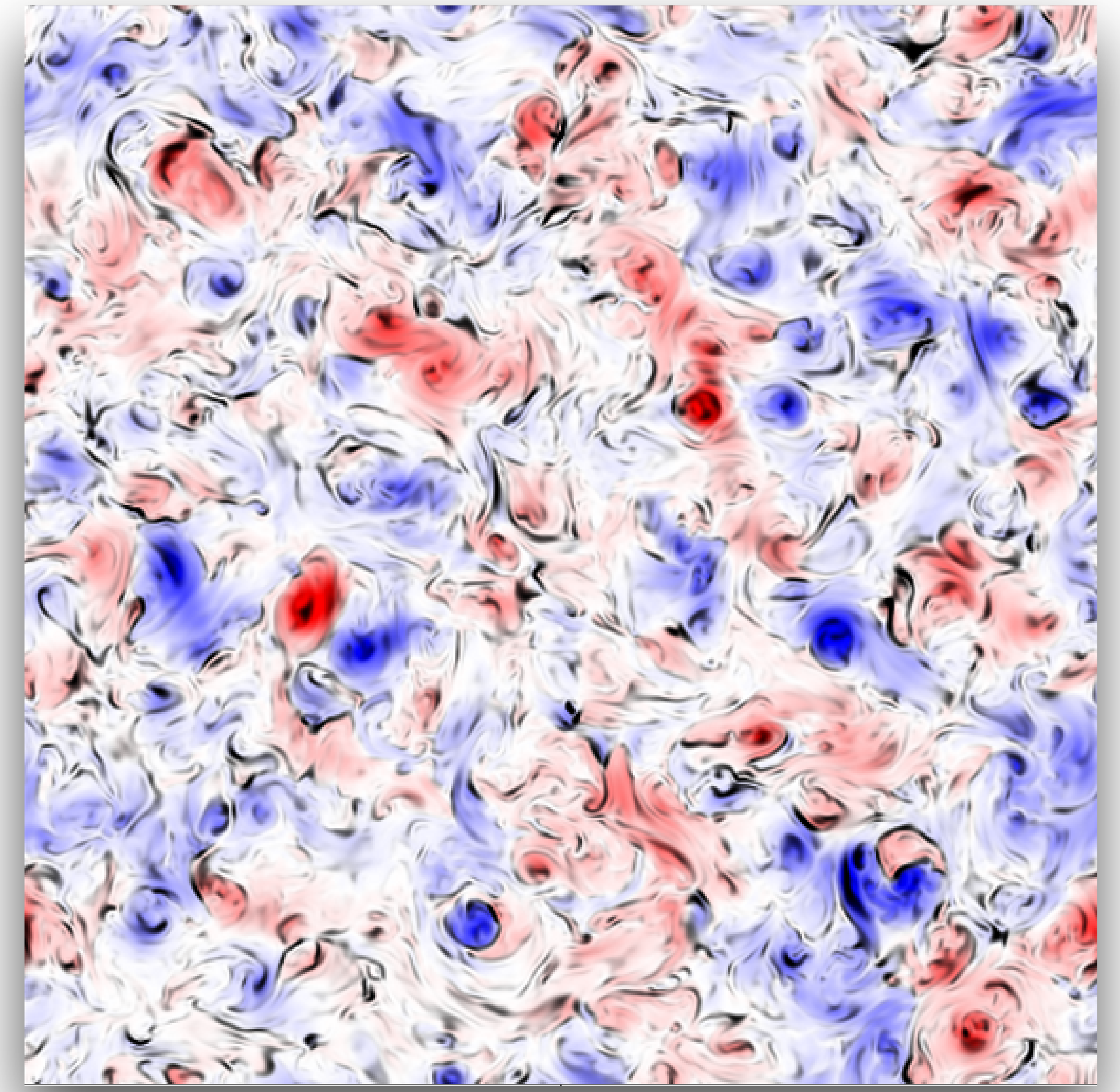
$$\frac{\partial}{\partial t}\langle h(\boldsymbol{x}_1)h(\boldsymbol{x}_2)\rangle + \frac{\partial}{\partial \boldsymbol{x}_1}\cdot\langle h(\boldsymbol{x}_2)\boldsymbol{F}(\boldsymbol{x}_1)\rangle + \frac{\partial}{\partial \boldsymbol{x}_2}\cdot\langle h(\boldsymbol{x}_1)\boldsymbol{F}(\boldsymbol{x}_2)\rangle = 0$$



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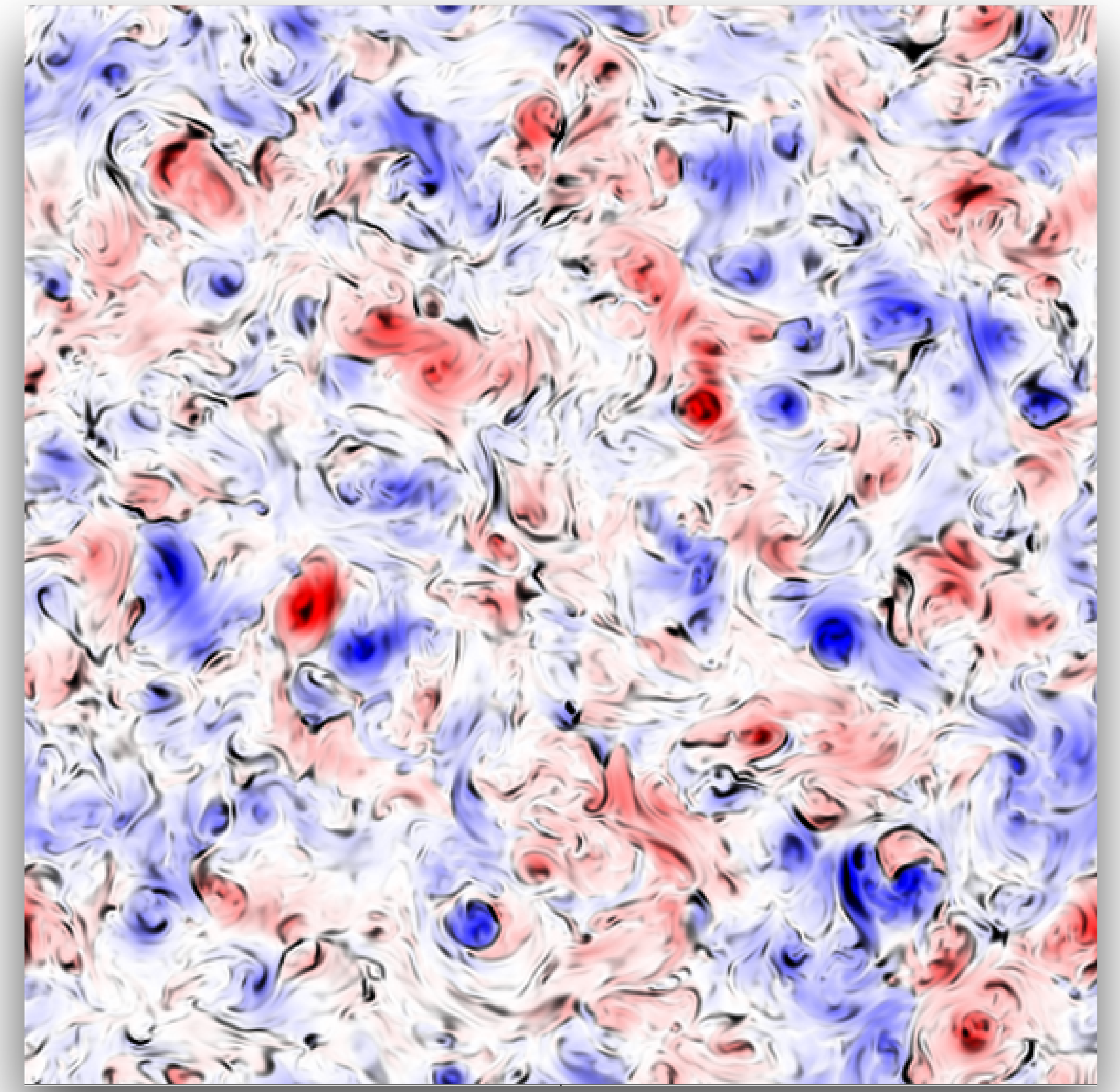
$$\frac{\partial}{\partial t}\langle h(\boldsymbol{x}_1) h(\boldsymbol{x}_2)\rangle + \underbrace{\frac{\partial}{\partial \boldsymbol{x}_1}}_{-\frac{\partial}{\partial \boldsymbol{r}}} \cdot \underbrace{\langle h(\boldsymbol{x}_2) \boldsymbol{F}(\boldsymbol{x}_1)\rangle}_{-\psi(r)\boldsymbol{r}} + \underbrace{\frac{\partial}{\partial \boldsymbol{x}_2}}_{\frac{\partial}{\partial \boldsymbol{r}}} \cdot \underbrace{\langle h(\boldsymbol{x}_1) \boldsymbol{F}(\boldsymbol{x}_2)\rangle}_{\psi(r)\boldsymbol{r}} = 0$$

$$\boldsymbol{x}_2 - \boldsymbol{x}_1 \equiv \boldsymbol{r}, \;\; \langle h(\boldsymbol{x}_1) \boldsymbol{F}(\boldsymbol{x}_1 + \boldsymbol{r})\rangle = \psi(r)\boldsymbol{r}$$



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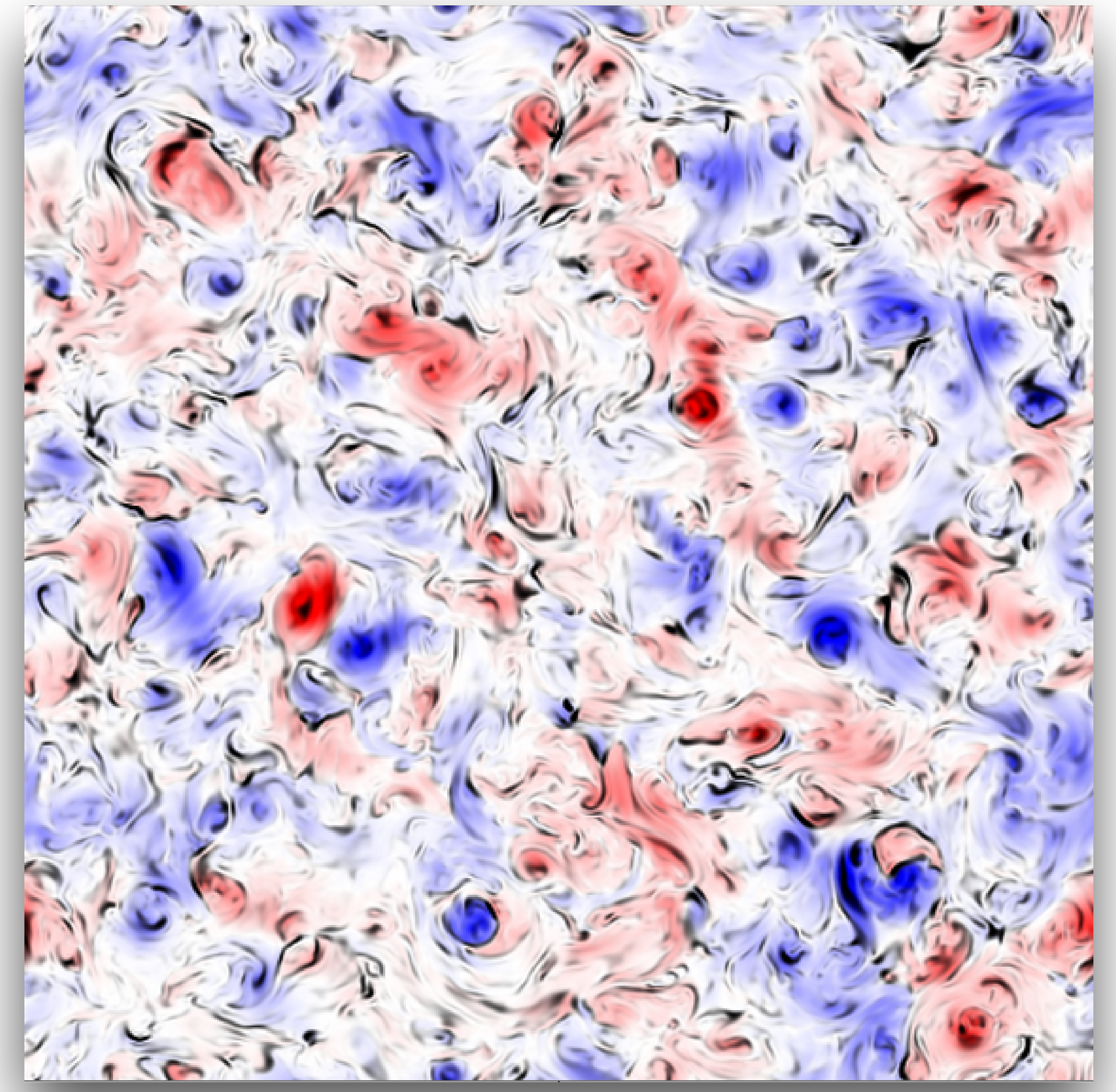
$$\frac{\partial}{\partial t}\langle h(\boldsymbol{x}) h(\boldsymbol{x}+\boldsymbol{r})\rangle + \frac{2}{r^2}\frac{\mathrm{d}}{\mathrm{d}r} r^3 \psi = 0$$

$$I_H \equiv \int \mathrm{d}^3\boldsymbol{r} \langle h(\boldsymbol{x}) h(\boldsymbol{x}+\boldsymbol{r})\rangle \qquad \frac{\mathrm{d}I_H}{\mathrm{d}t} = -8\pi \lim_{r\to\infty} r^3 \psi$$

Thus, I_H is conserved by homogeneous isotropic turbulence with spatial correlations that decay sufficiently fast.



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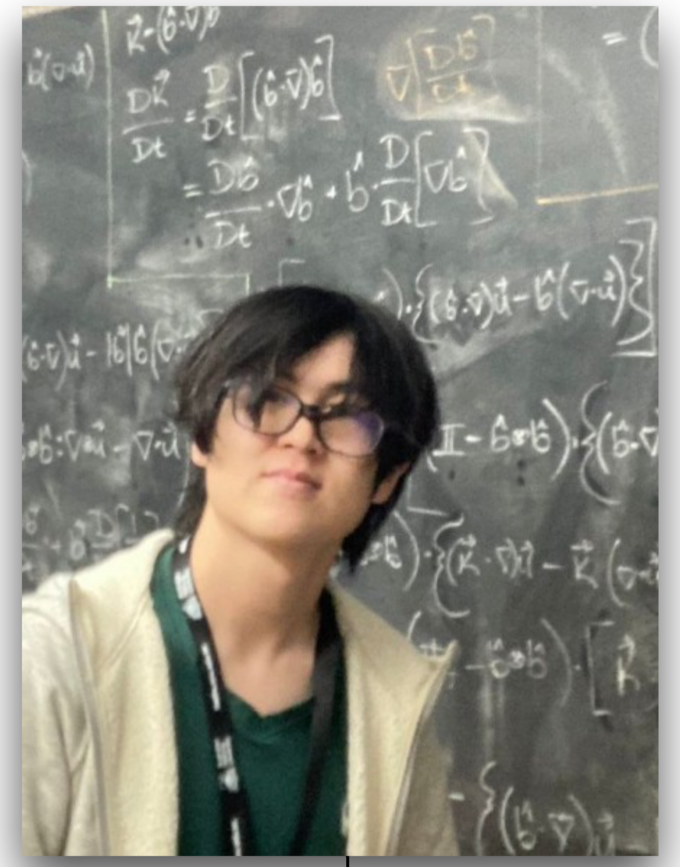

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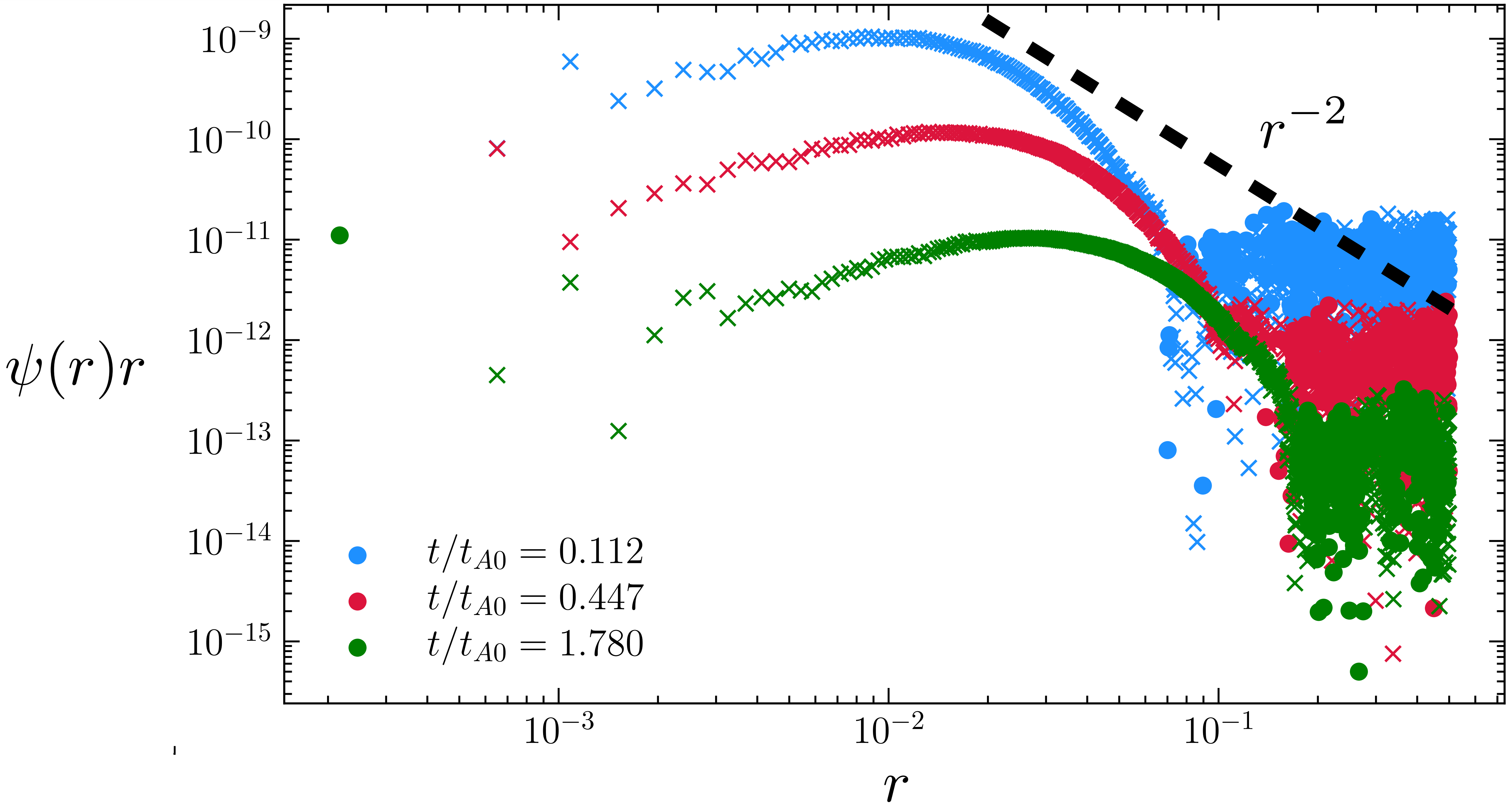
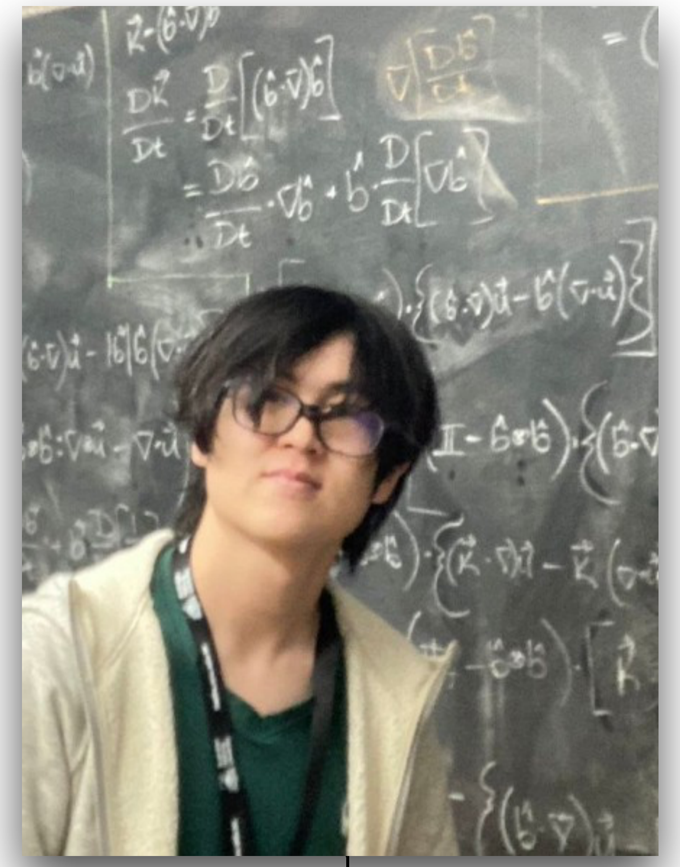


J. Kin Jun Hew, D. N. Hosking, C. Federrath et. al., *in prep.*



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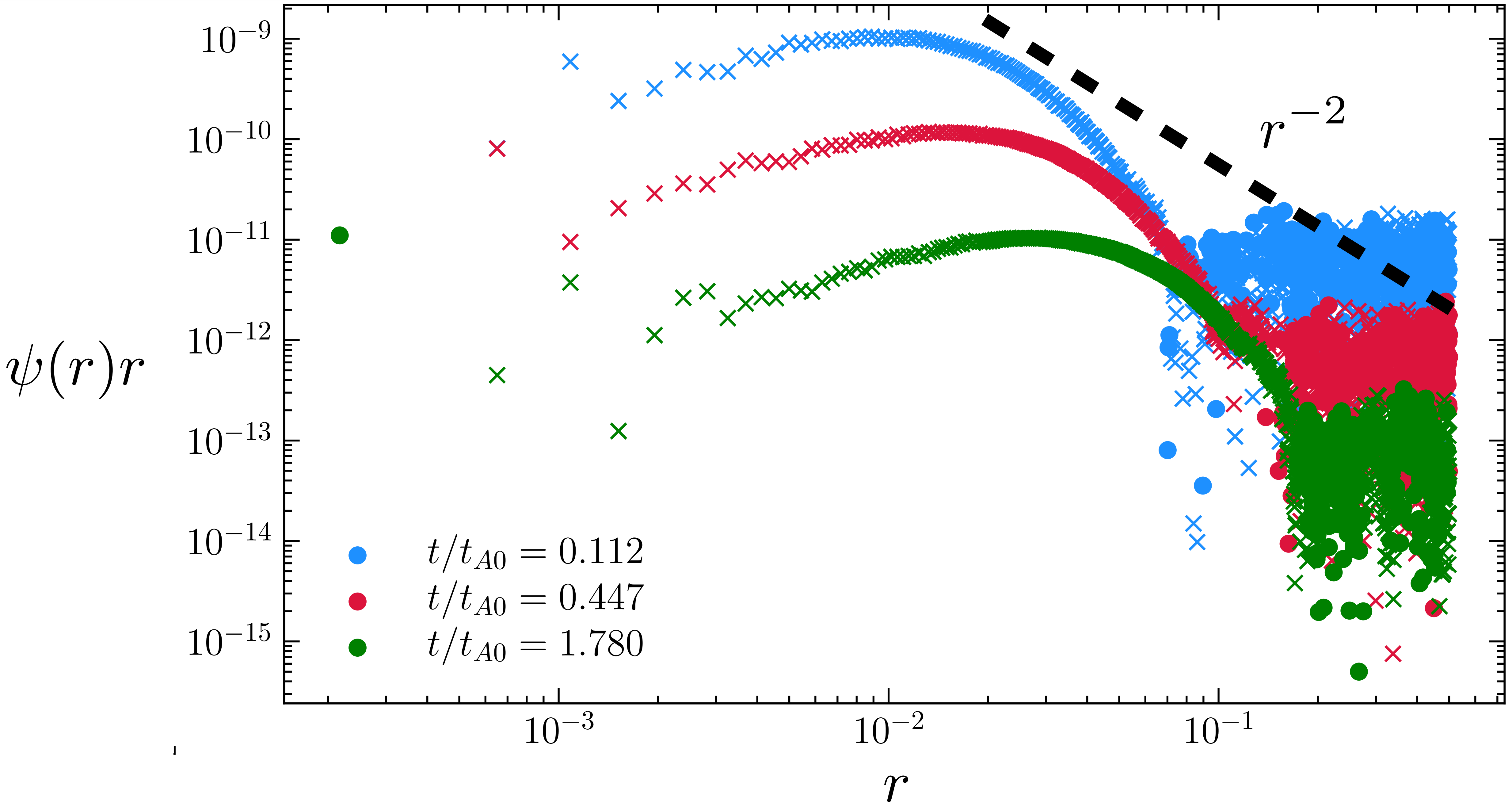

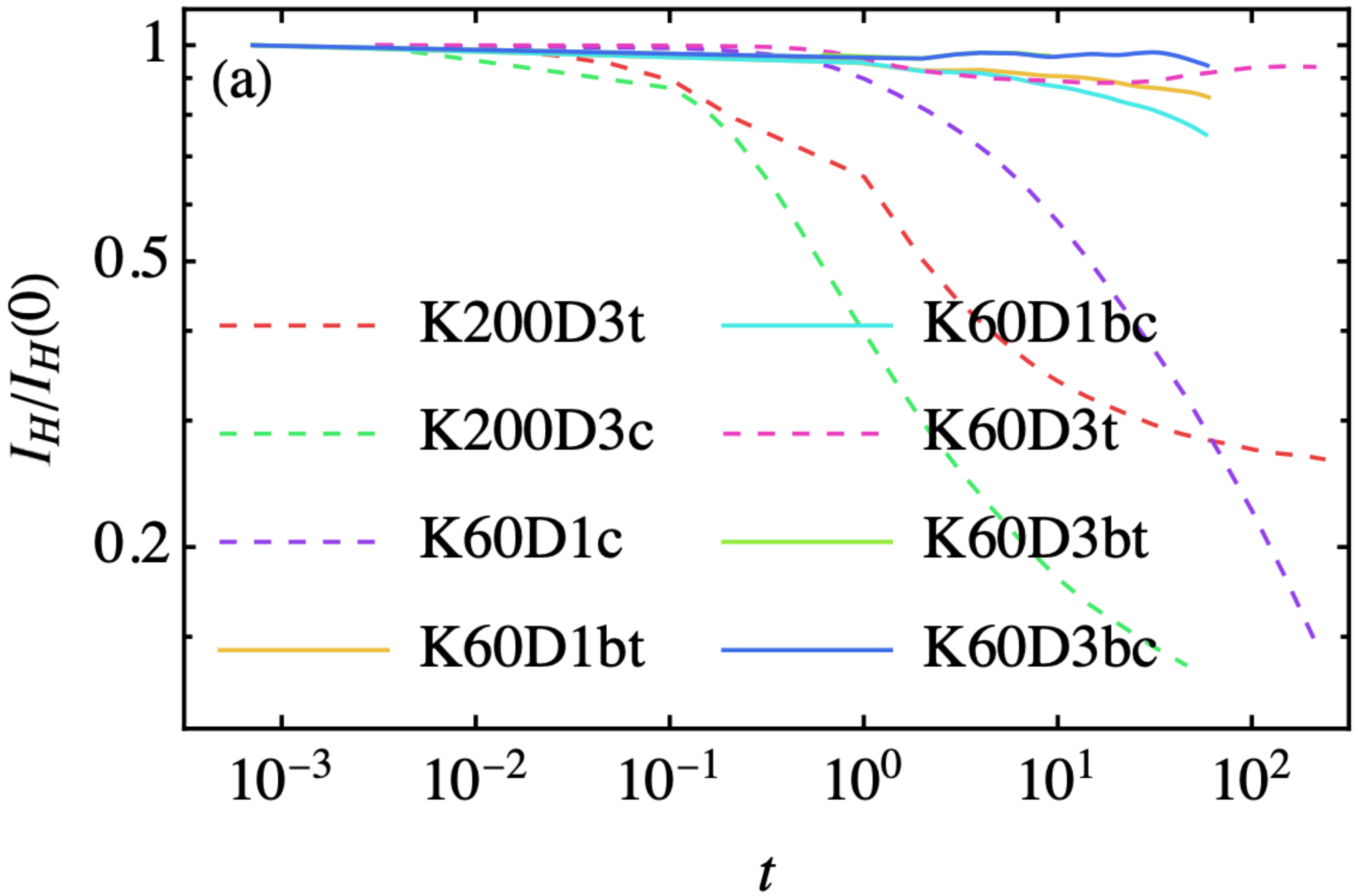

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H. Zhou et. al. 2022

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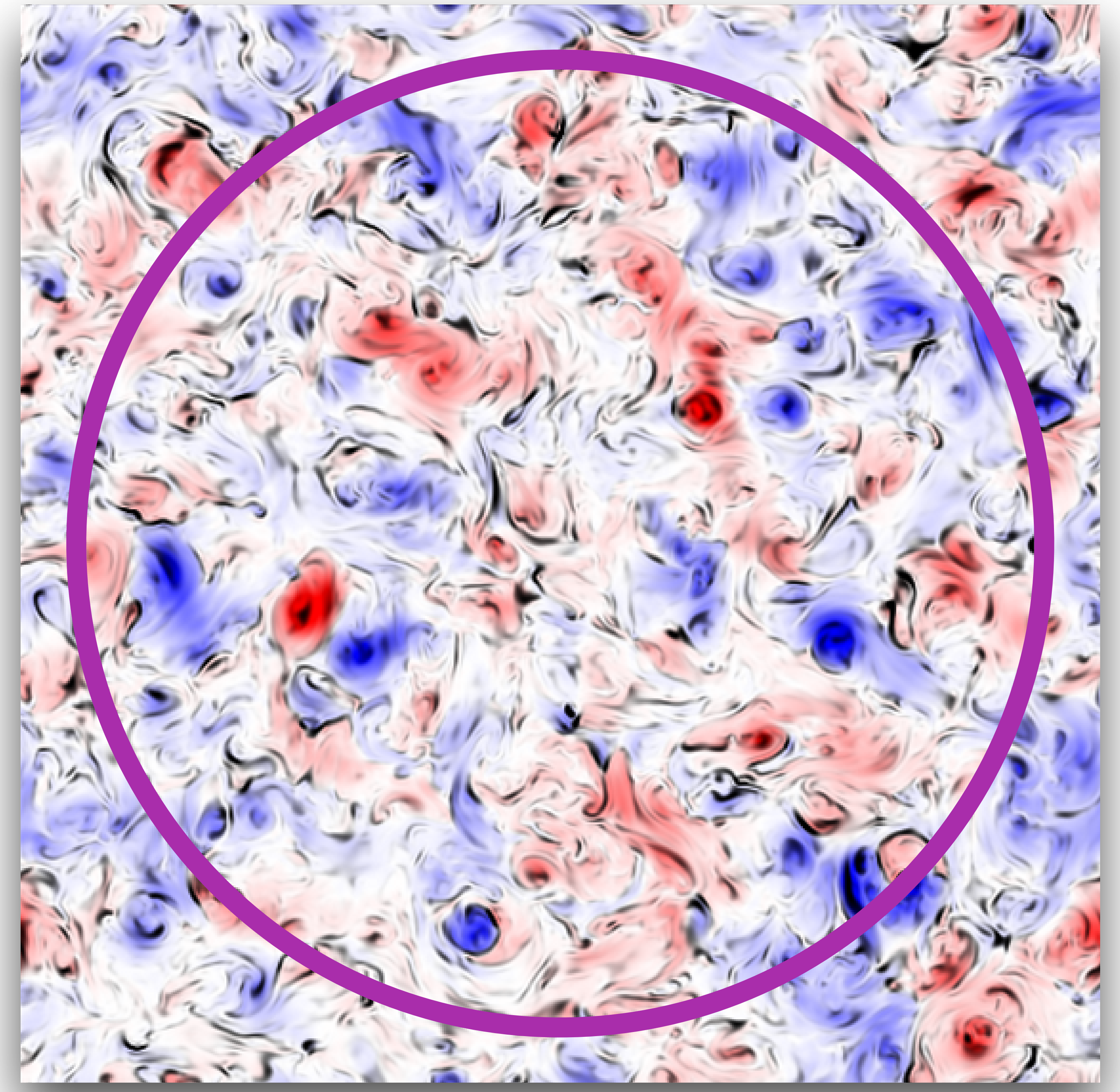

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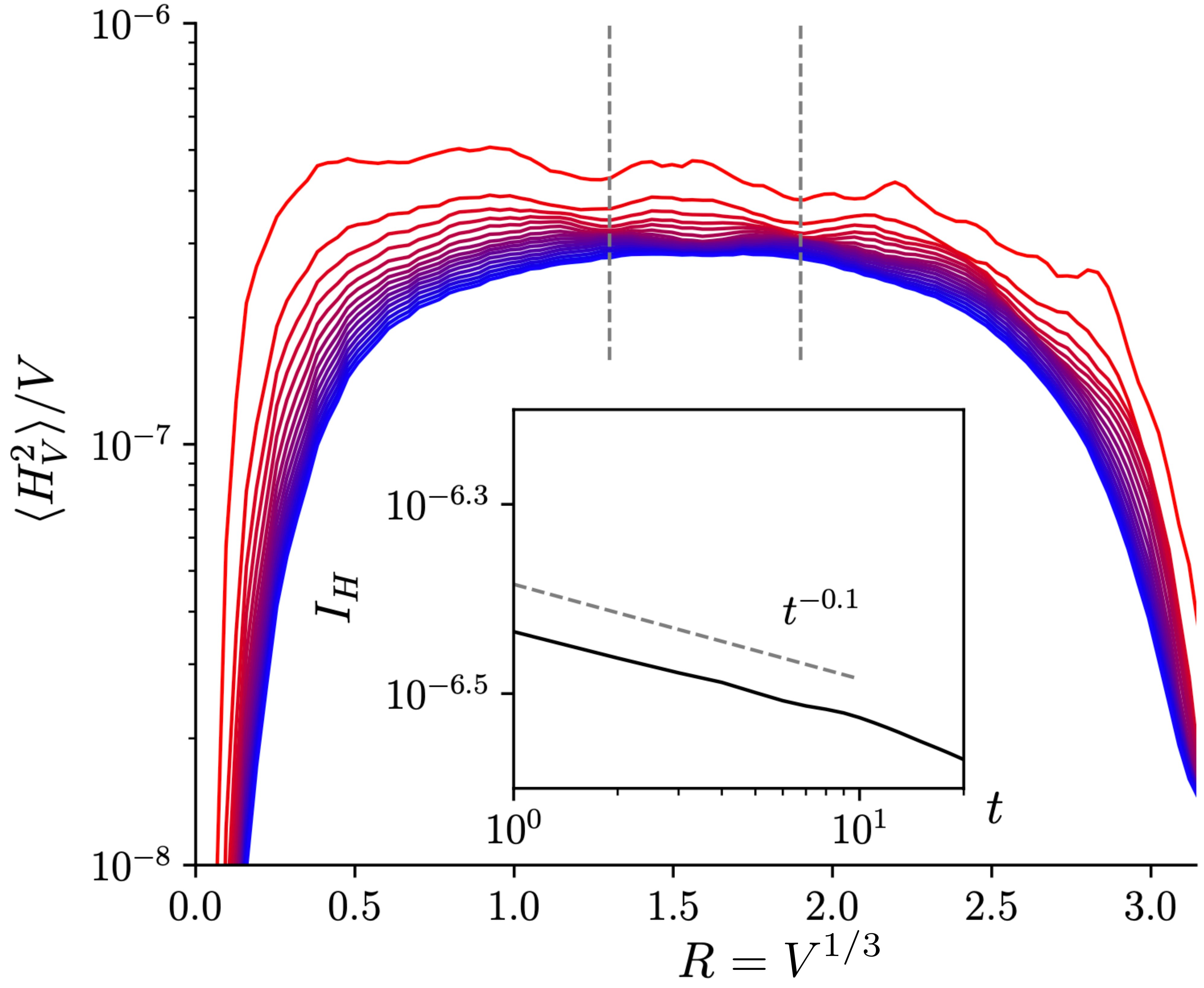
$$I_H = \lim_{V\to\infty}\frac{1}{V}H_V^2$$

$$H_V^2 \propto V$$

$$\frac{\mathrm{d}H_V^2}{\mathrm{d}t} \propto V^{2/3}$$



Hosking & Schekochihin 2021

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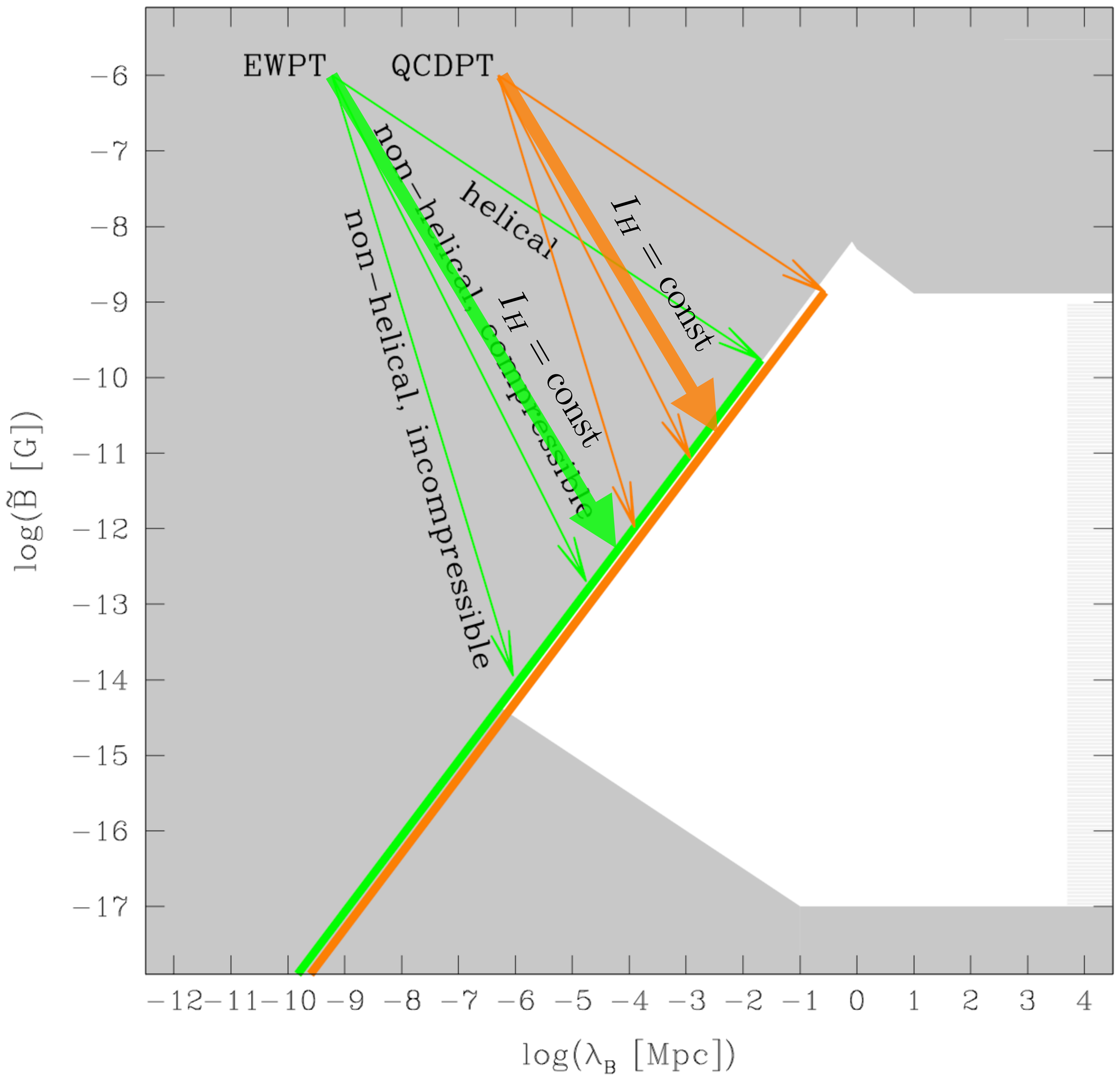
$$I_H \equiv \int \mathrm{d}^3\boldsymbol{r} \langle h(\boldsymbol{x}) h(\boldsymbol{x}+\boldsymbol{r}) \rangle \sim B^4 \lambda_B^5$$



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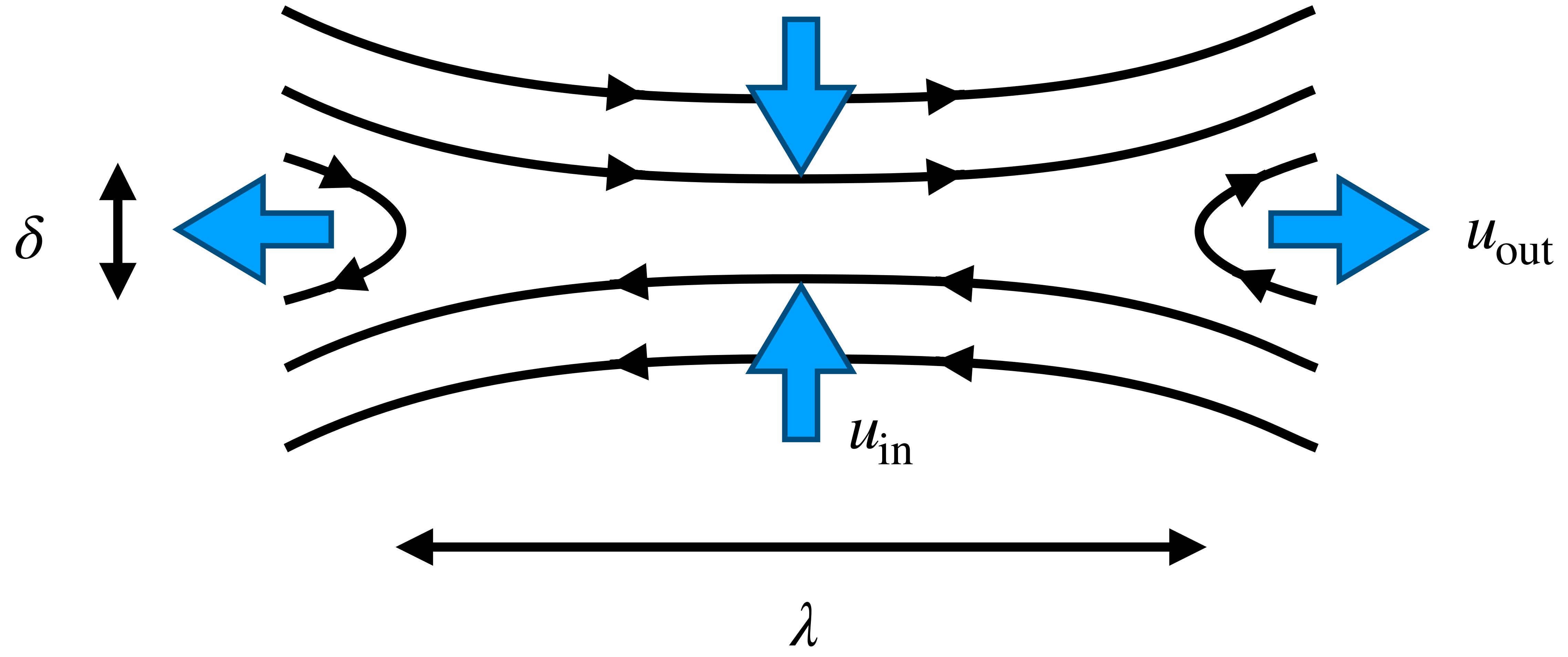
Sweet-Parker reconnection



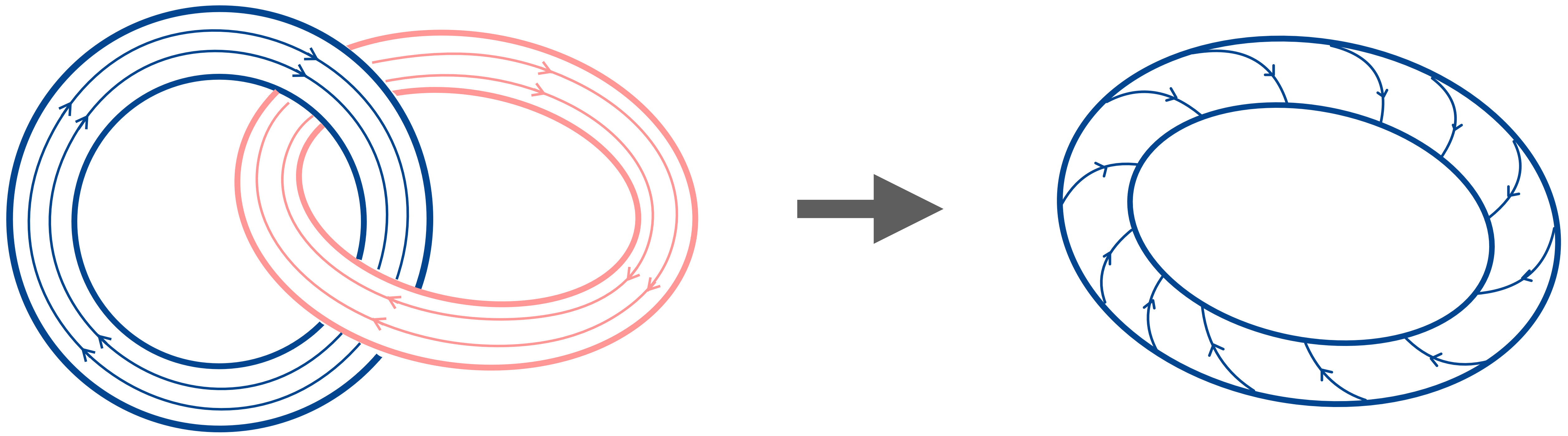
Induction: $\nabla \times (\boldsymbol{u} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{B} = 0 \implies u_{in} \sim \frac{\eta}{\delta}$

Continuity: $\nabla \cdot \boldsymbol{u} = 0 \implies u_{\text{in}} \lambda \sim u_{\text{out}} \delta$

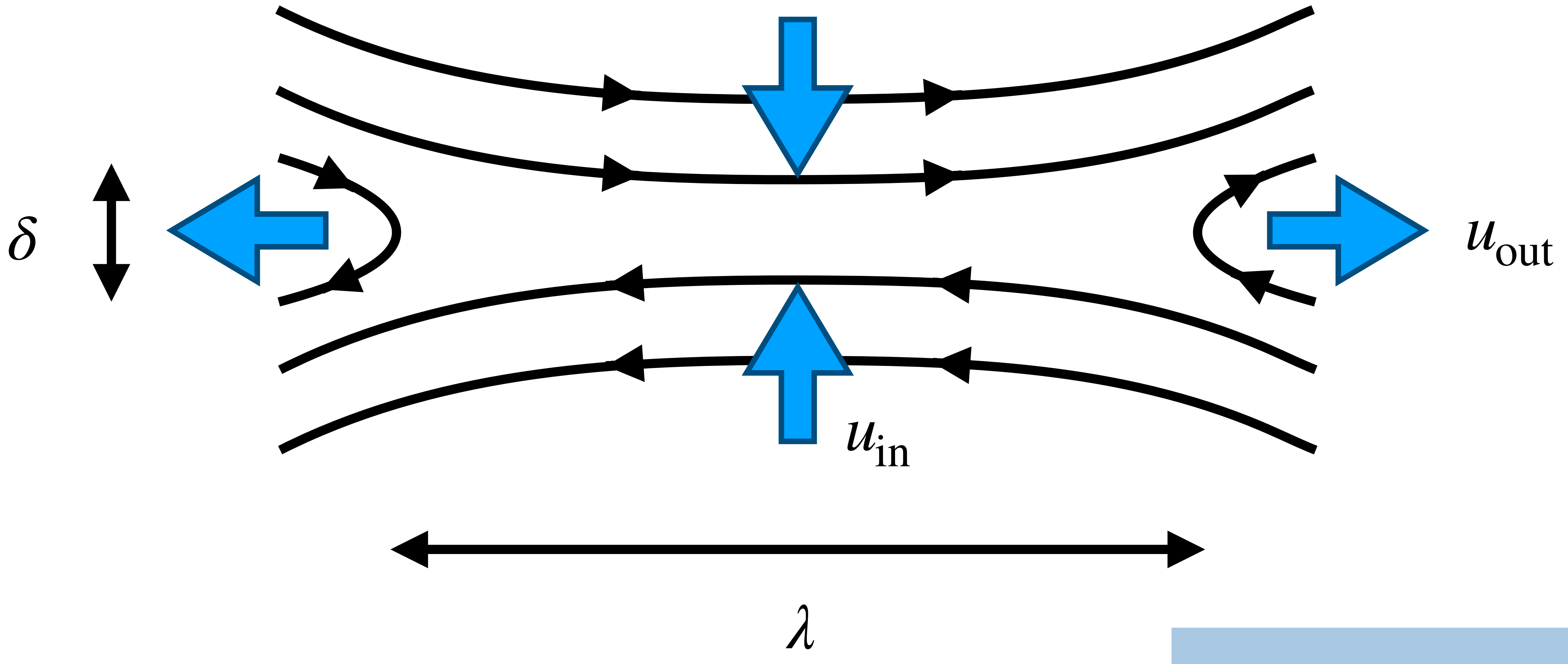
Momentum: $\boldsymbol{u} \cdot \nabla \boldsymbol{u} \sim \boldsymbol{B} \cdot \nabla \boldsymbol{B} \implies u_{\text{out}} \sim v_A$

$$u_{\text{out}} \sim v_A, \quad \frac{u_{\text{in}}}{u_{\text{out}}} \sim S^{-1/2}, \quad \frac{\delta}{\lambda} \sim S^{-1/2}, \quad S \equiv \frac{v_A \lambda}{\eta}$$

Sweet (1956), Parker (1957)



Sweet-Parker reconnection



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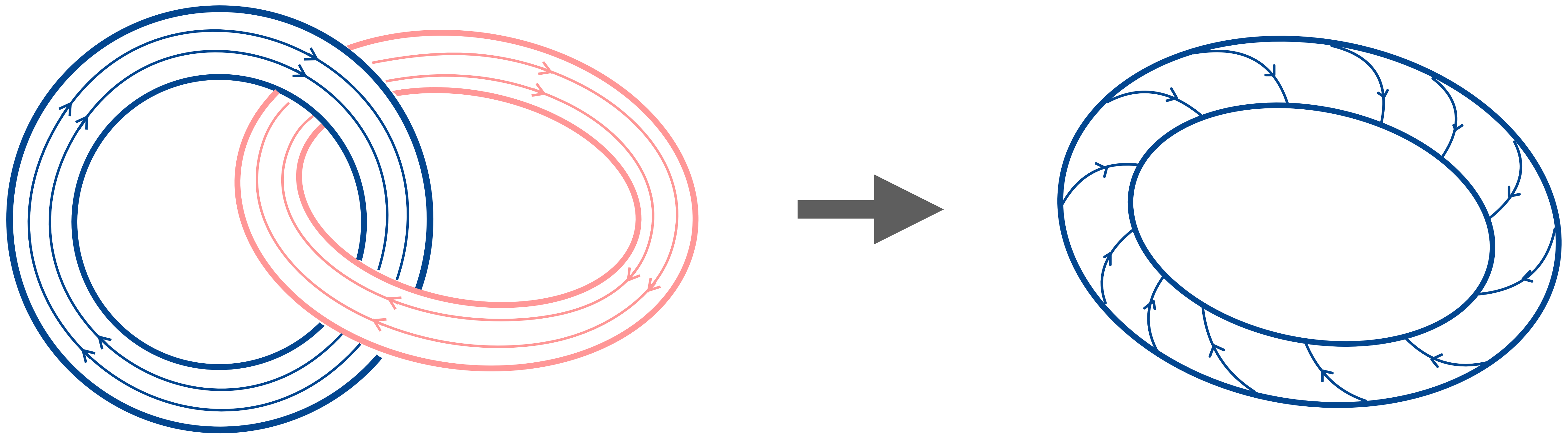
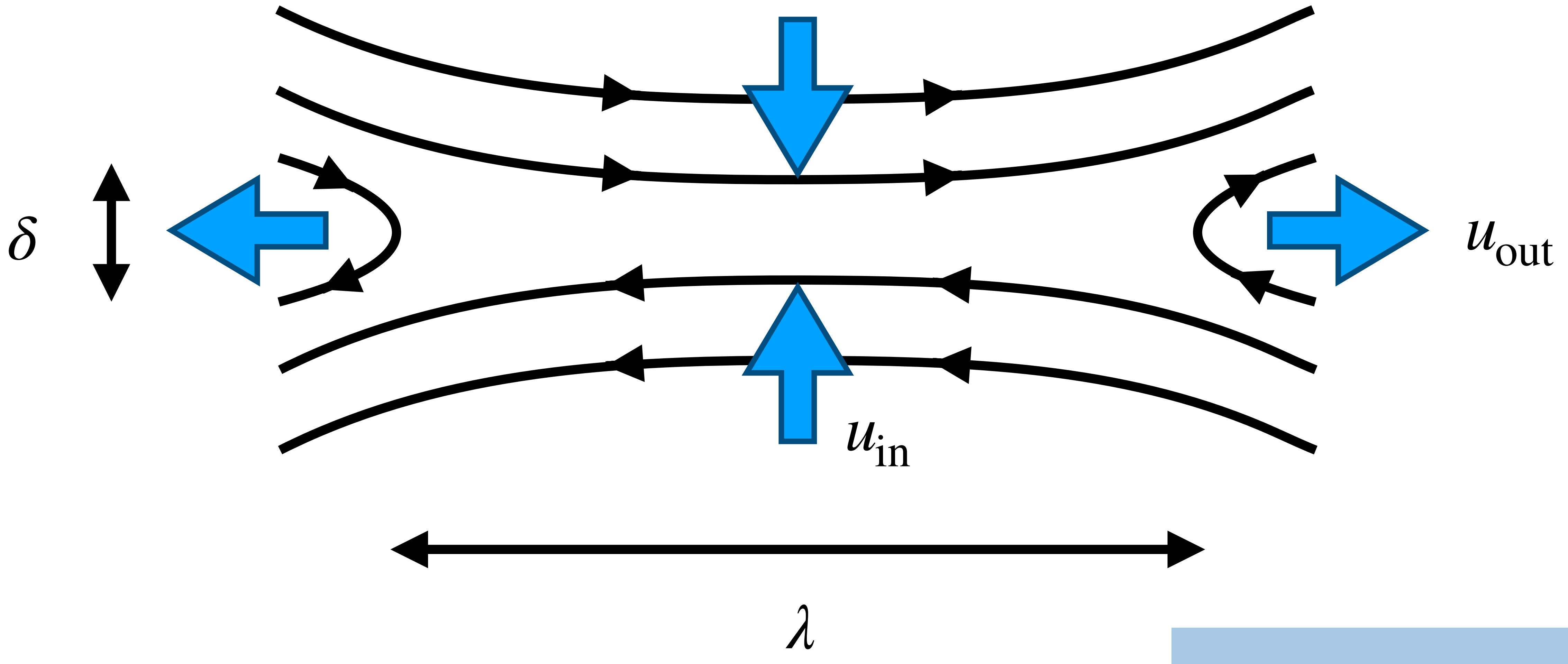
$$\implies \frac{1}{\tau_{\text{rec}}} \equiv \frac{u_{\text{in}}}{\lambda} \sim S^{-1/2} \frac{v_A}{\lambda}$$

Sweet (1956), Parker (1957)



Sweet-Parker reconnection



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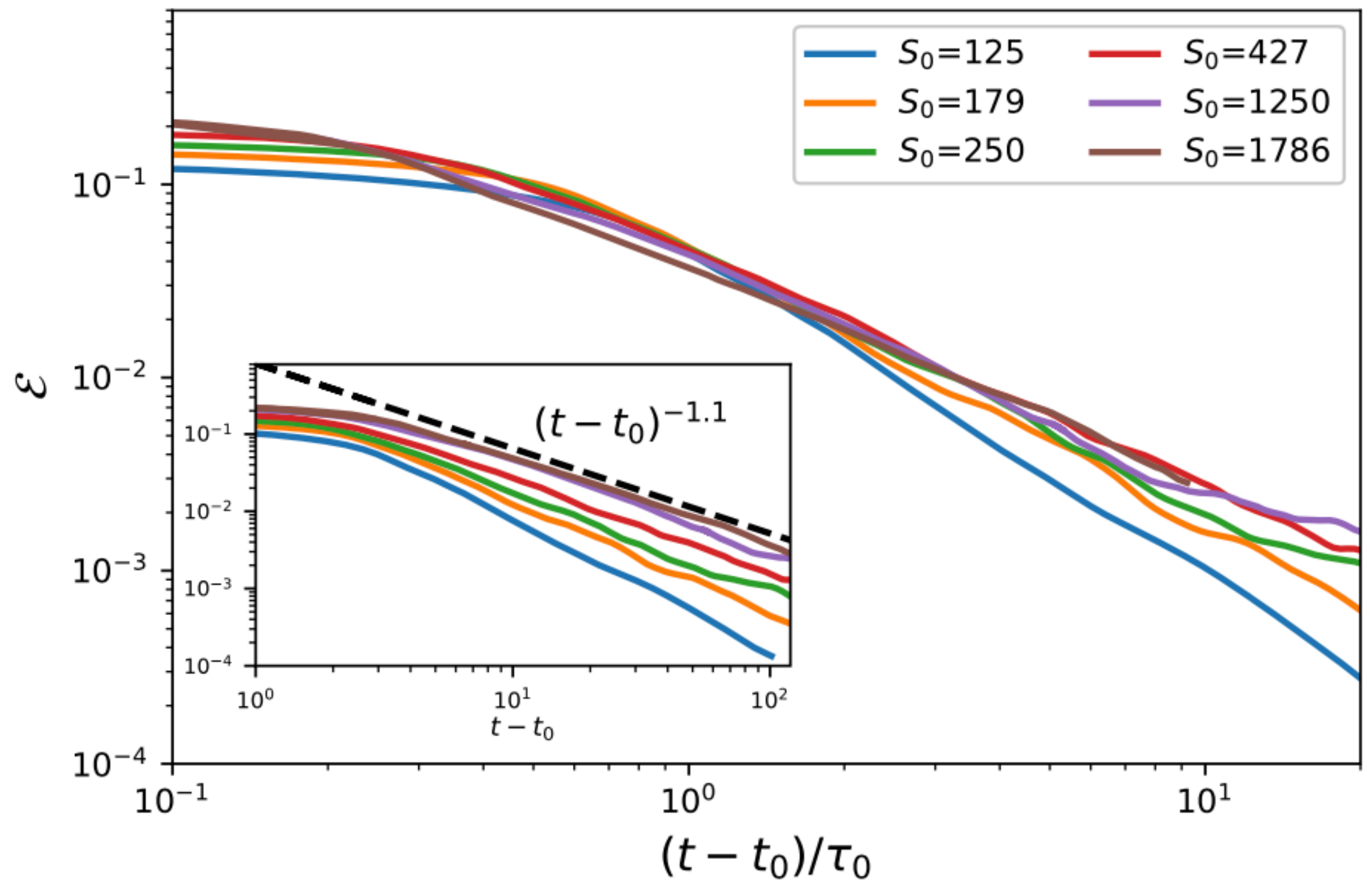
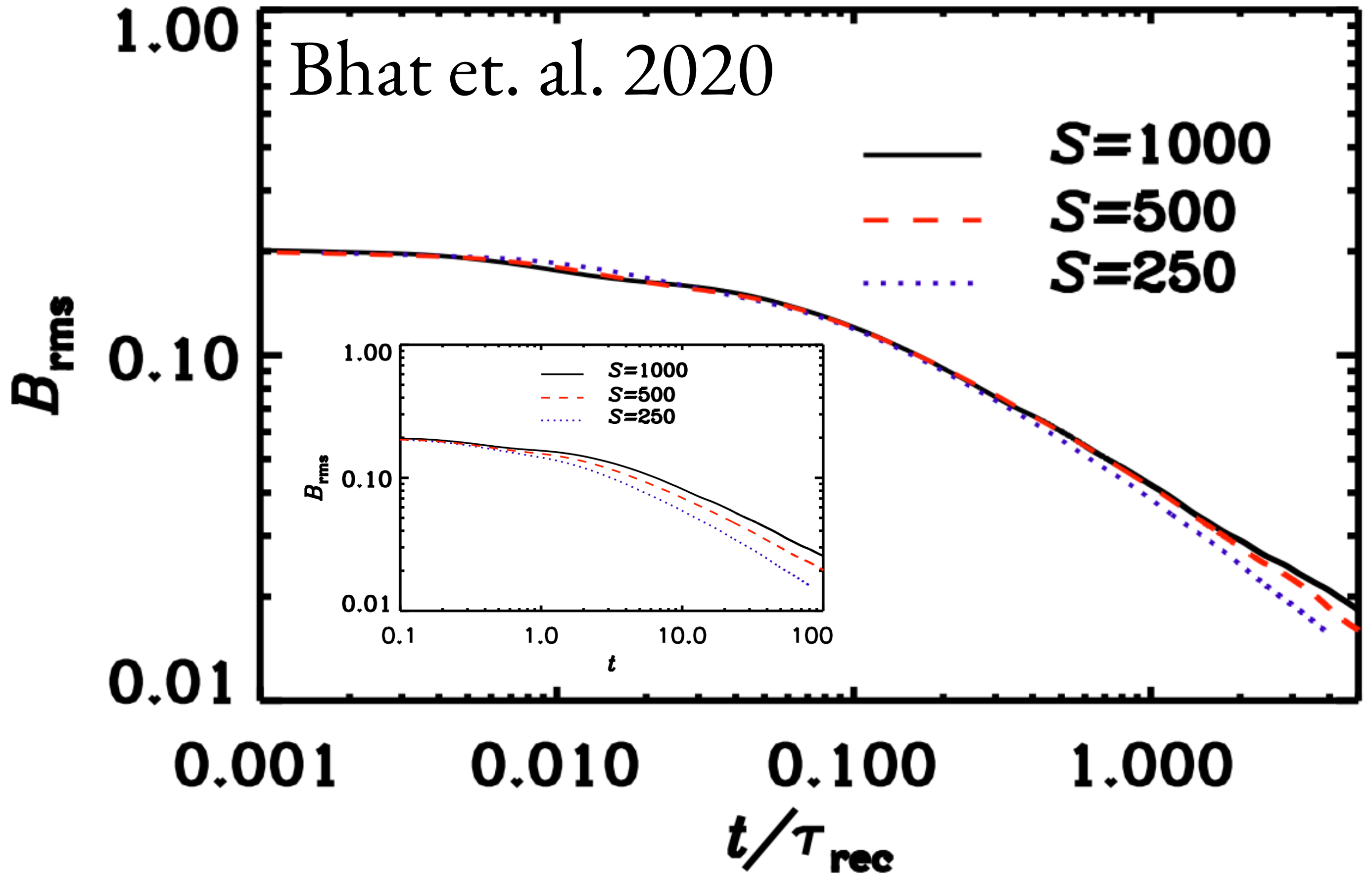
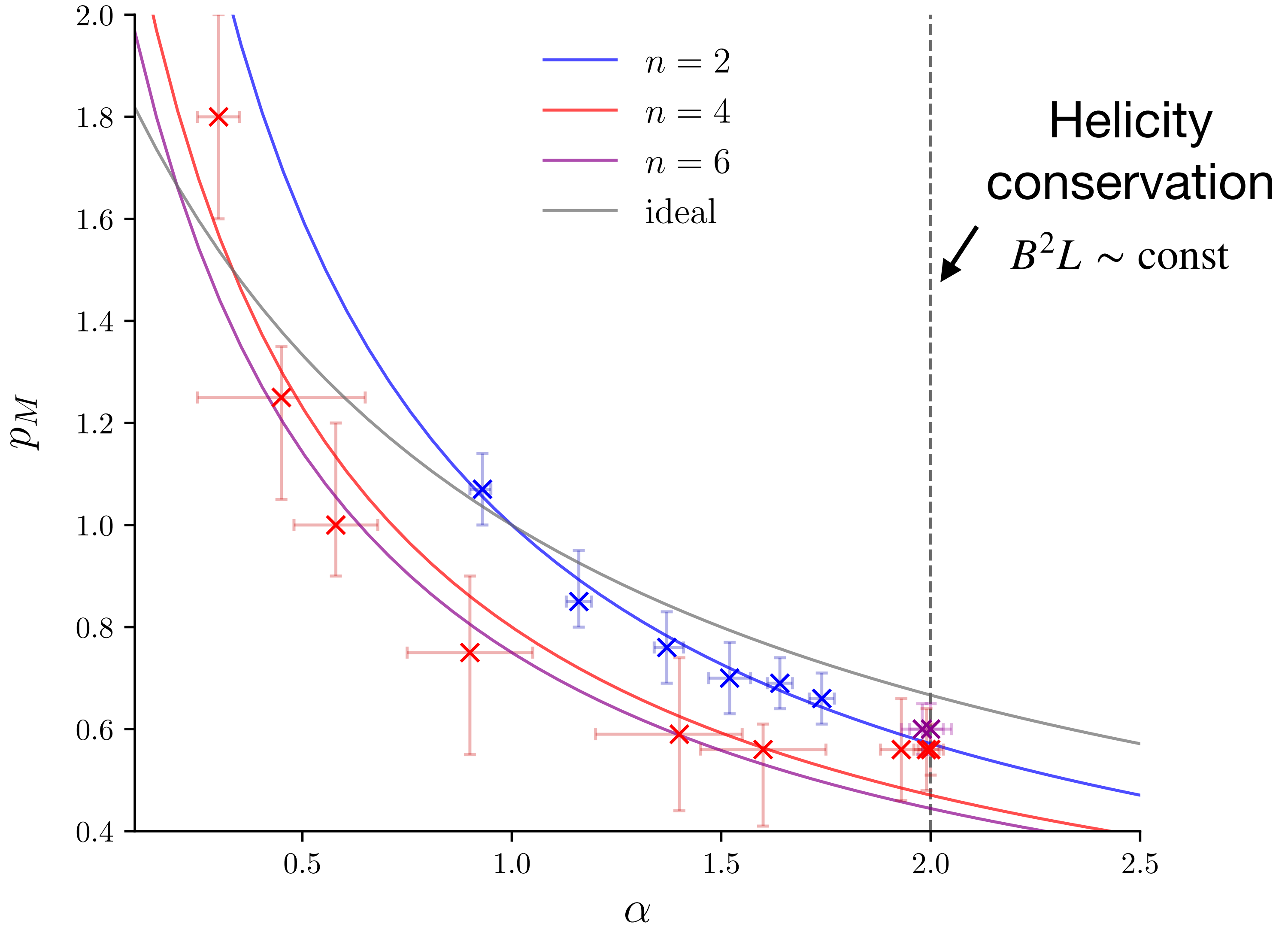
M. Zhou et. al. 2019

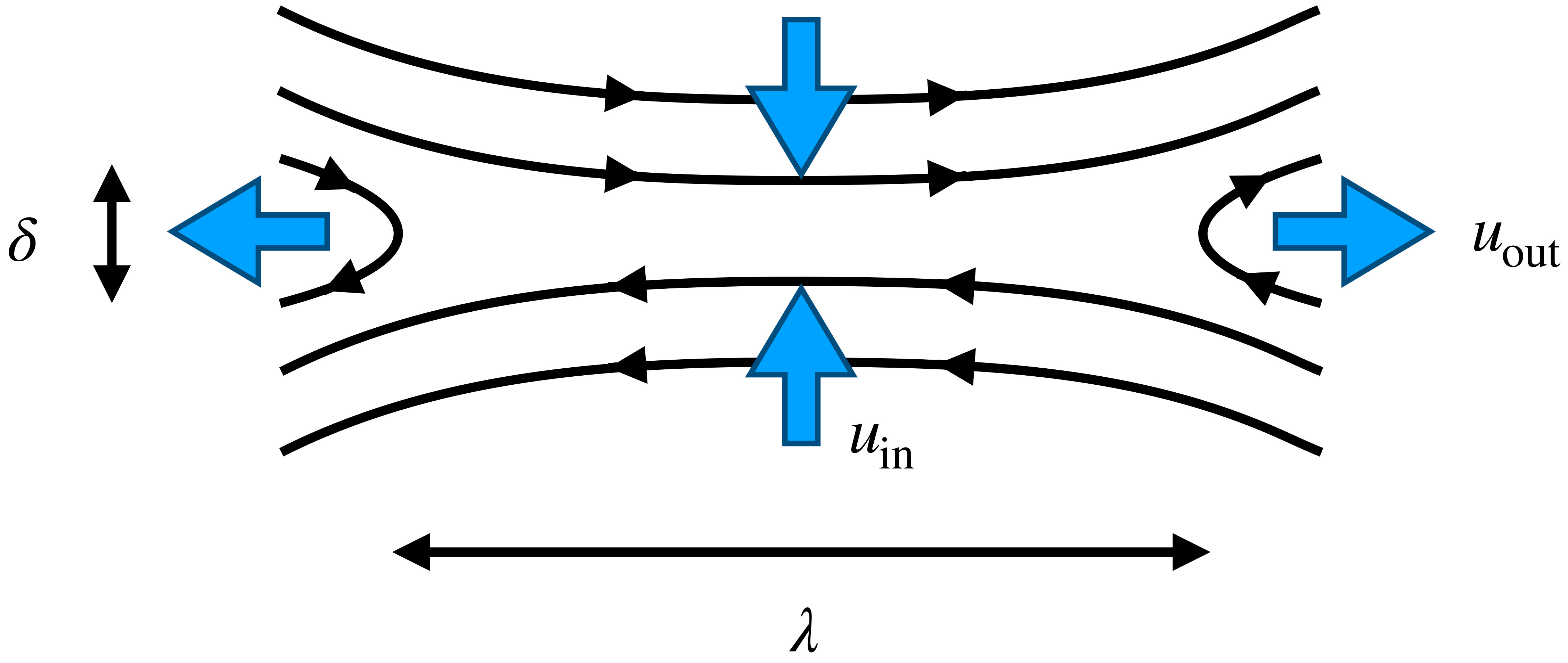




Hosking & Schekochihin 2021



Sweet-Parker reconnection



Induction: $\nabla \times (\boldsymbol{u} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{B} = 0 \implies u_{in} \sim \frac{\eta}{\delta}$

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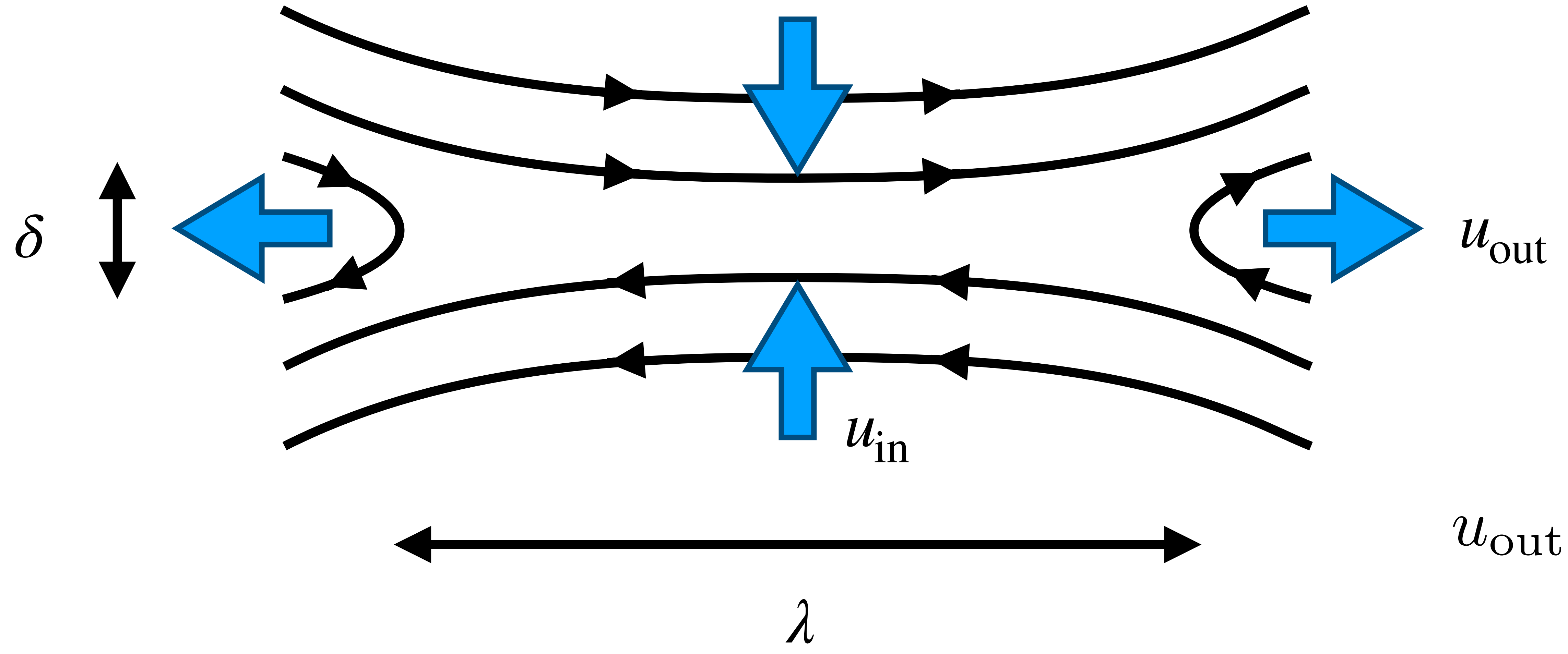
As $\eta \to 0$, $\quad \dfrac{\boldsymbol{u} \cdot \nabla \boldsymbol{u}}{\nu \nabla^2 \boldsymbol{u}} \sim \dfrac{u_{\text{out}}/\lambda}{\nu/\delta^2} \sim \dfrac{1}{\text{Pm}}$

Thus, for Pm $\gg 1$, we should balance the Lorentz force with viscosity rather than inertia (Park et. al. 1984).

Sweet-Parker reconnection



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Momentum: $\nu \nabla^2 \boldsymbol{u} \sim \boldsymbol{B} \cdot \boldsymbol{\nabla} \boldsymbol{B} \implies u_{\rm out} \sim \frac{\lambda v_A^2}{\nu \delta^2}$

$$u_{\rm out} \sim \frac{v_A}{\sqrt{1 + {\rm Pm}}}, \quad \frac{u_{\rm in}}{u_{\rm out}} \sim S^{-1/2}, \quad \frac{\delta}{\lambda} \sim S^{-1/2}, \quad S \equiv \frac{v_A \lambda}{\eta \sqrt{1 + {\rm Pm}}}$$

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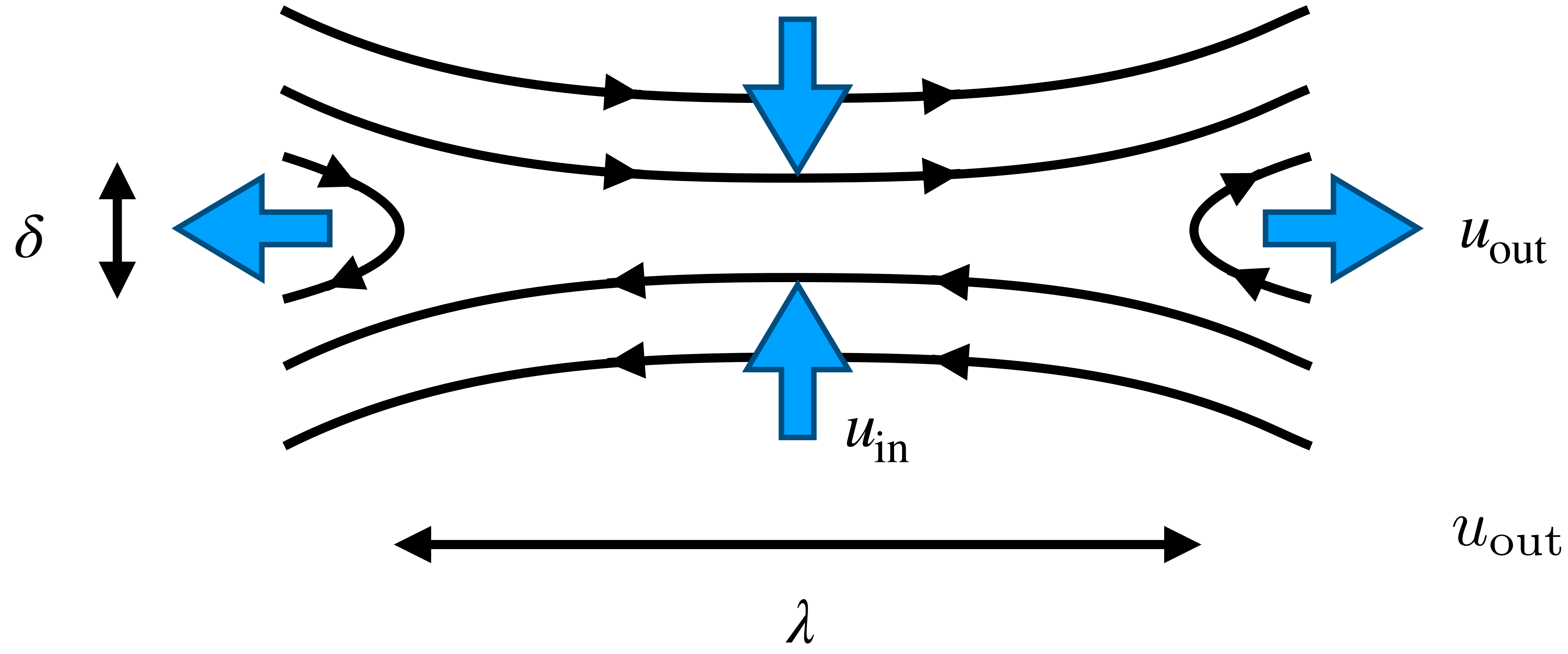
Thus, for Pm $\gg$ 1, we should balance the Lorentz force with viscosity rather than inertia (Park et. al. 1984).

The reconnection rate is then

$$\frac{1}{\tau_{\rm rec}} \equiv \frac{u_{\rm in}}{\lambda} \sim \frac{1}{S^{1/2}\sqrt{1 + {\rm Pm}}} \frac{v_A}{\lambda}$$

Sweet-Parker reconnection



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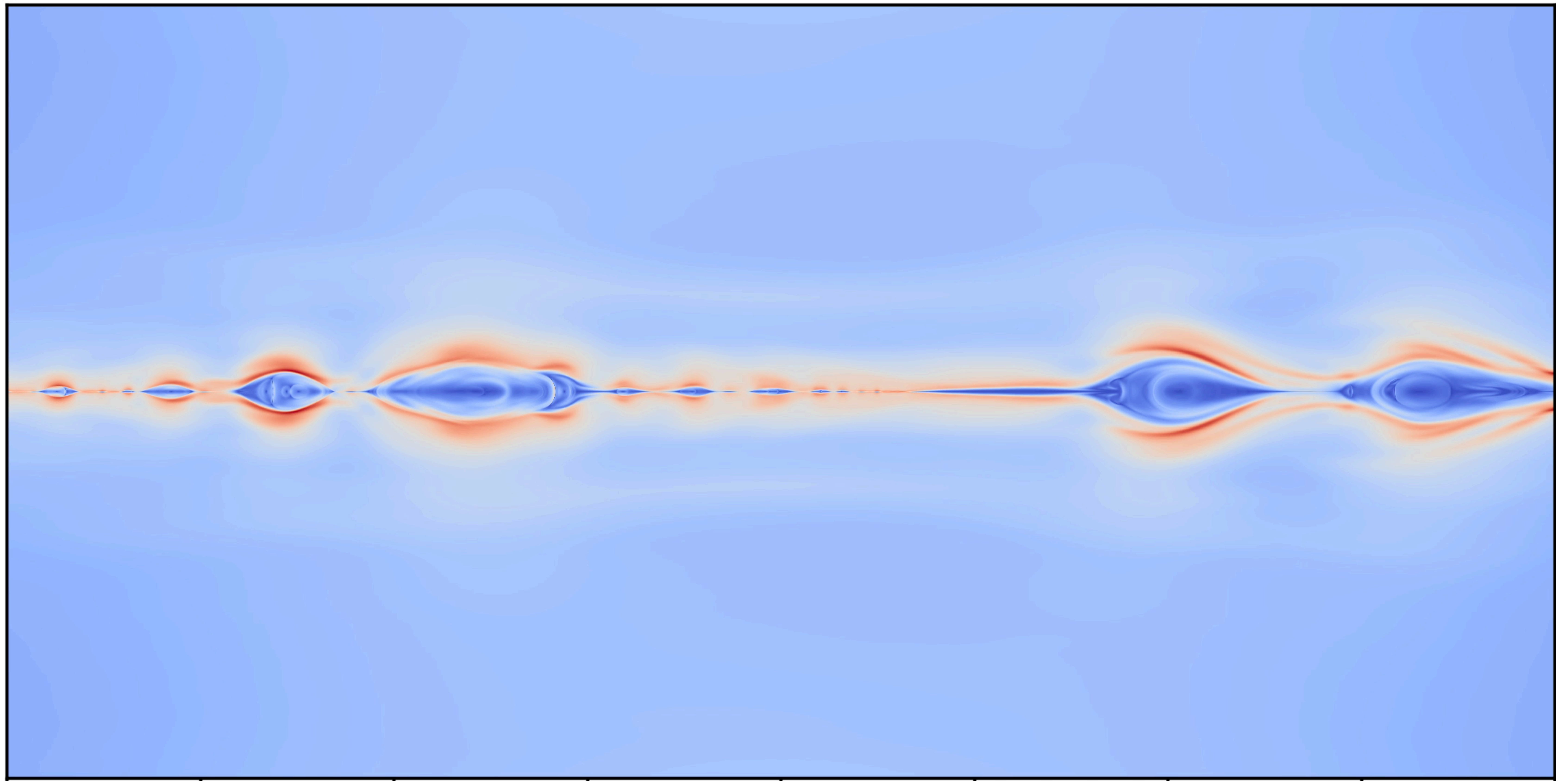
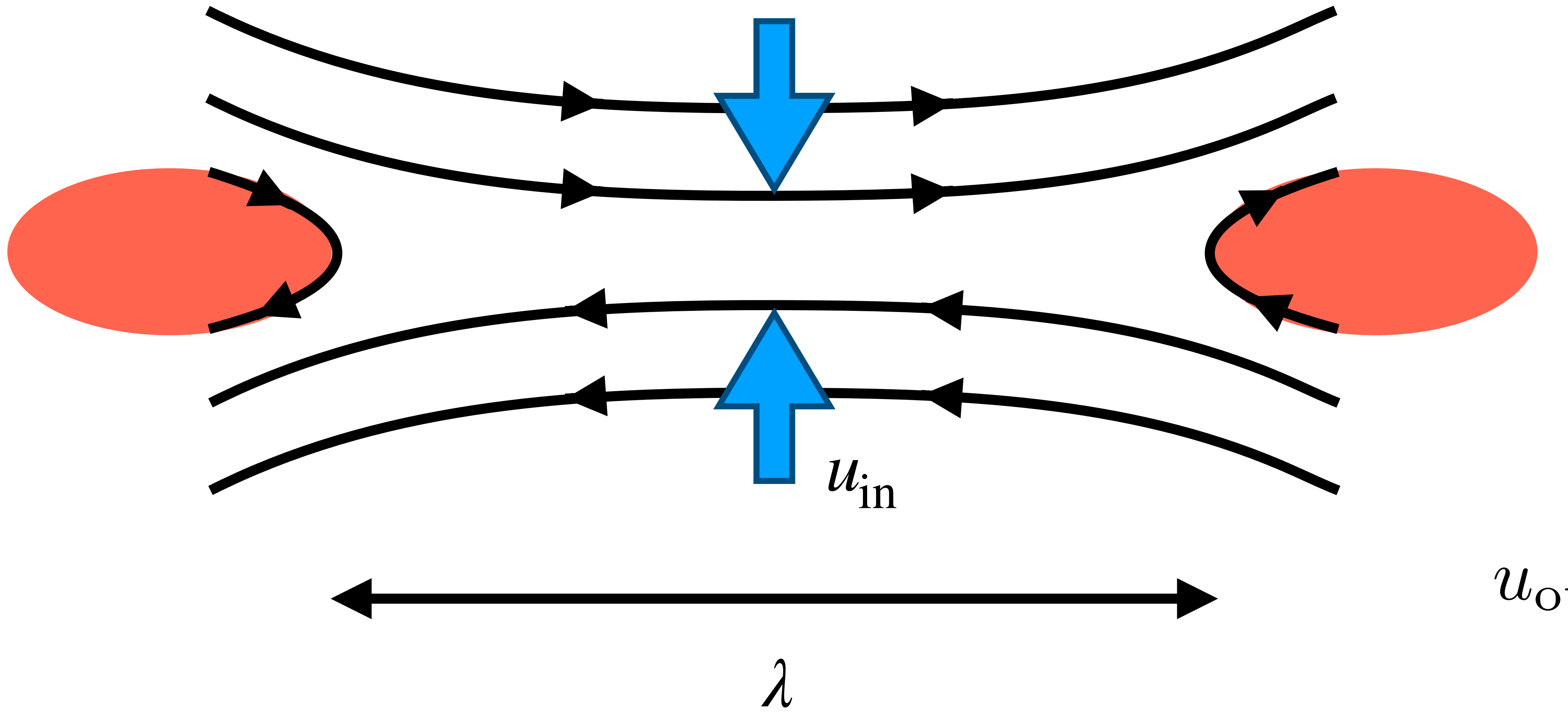
at recombination: $S \sim 10^9, \quad {\rm Pm} \sim 10^7 \implies \frac{1}{\tau_{\rm rec}} \sim 10^{-8} \frac{v_A}{\lambda}$

Plasmoid-mediated reconnection



Sweet-Parker sheets are unstable to the plasmoid instability (Loureiro 2007) for $S \gtrsim S_c \sim 10^4$.

Plasmoid-mediated reconnection



Induction: $\nabla \times (\boldsymbol{u} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{B} = 0 \implies u_{in} \sim \frac{\eta}{\delta}$

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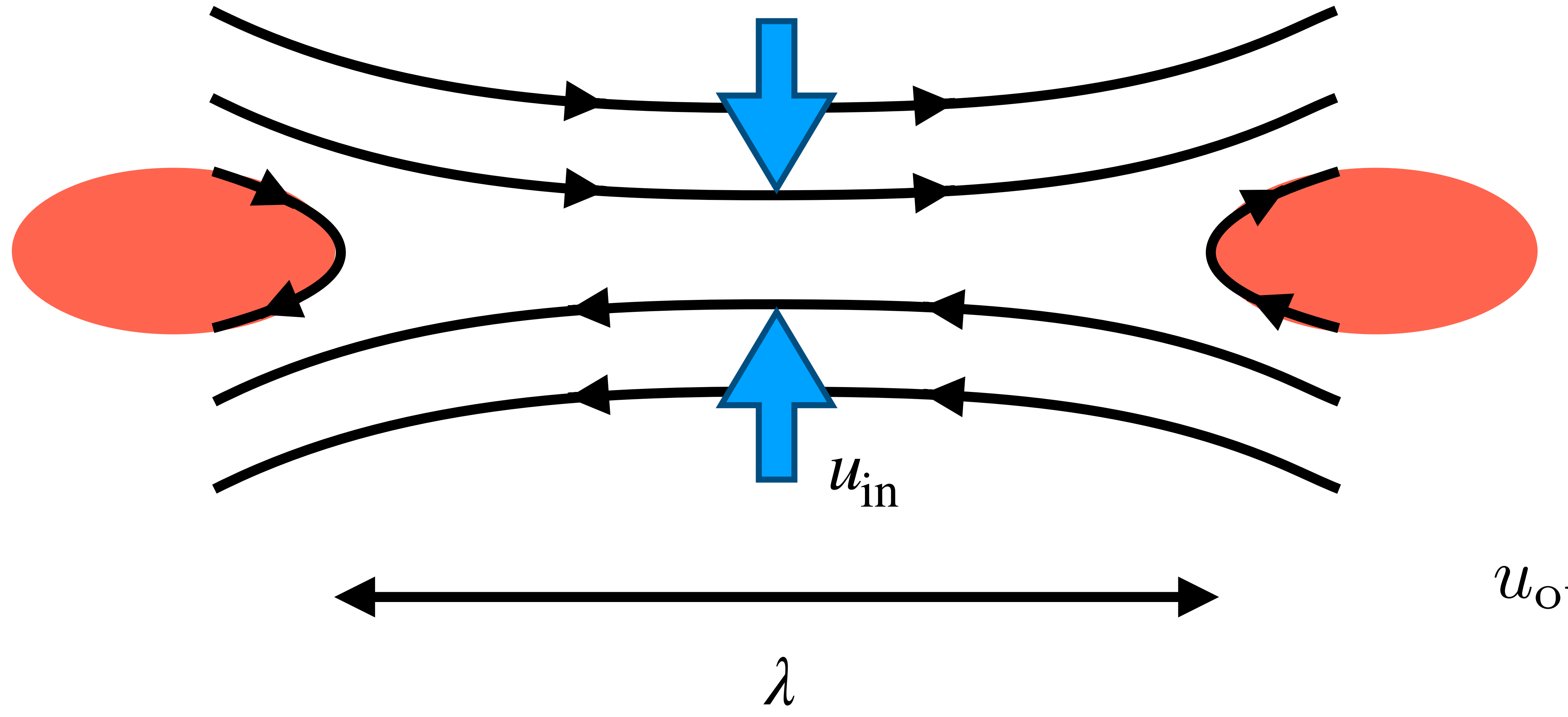
$$u_{\text{out}} \sim \frac{v_A}{\sqrt{1 + \text{Pm}}}, \quad \frac{u_{\text{in}}}{u_{\text{out}}} \sim S^{-1/2}, \quad \frac{\delta}{\lambda} \sim S^{-1/2}, \quad S \equiv \frac{v_A \lambda}{\eta \sqrt{1 + \text{Pm}}}$$

Between plasmoids, we have Sweet-Parker sheets. The inflow velocity is just the SP one with $S = S_c \sim 10^4$.

The reconnection rate is then (Uzdensky et. al. 2010)

$$\frac{1}{\tau_{\text{rec}}} \equiv \frac{u_{\text{in}}}{\lambda} \sim \frac{1}{S_c^{1/2}\sqrt{1 + \text{Pm}}} \frac{v_A}{\lambda}$$

Plasmoid-mediated reconnection



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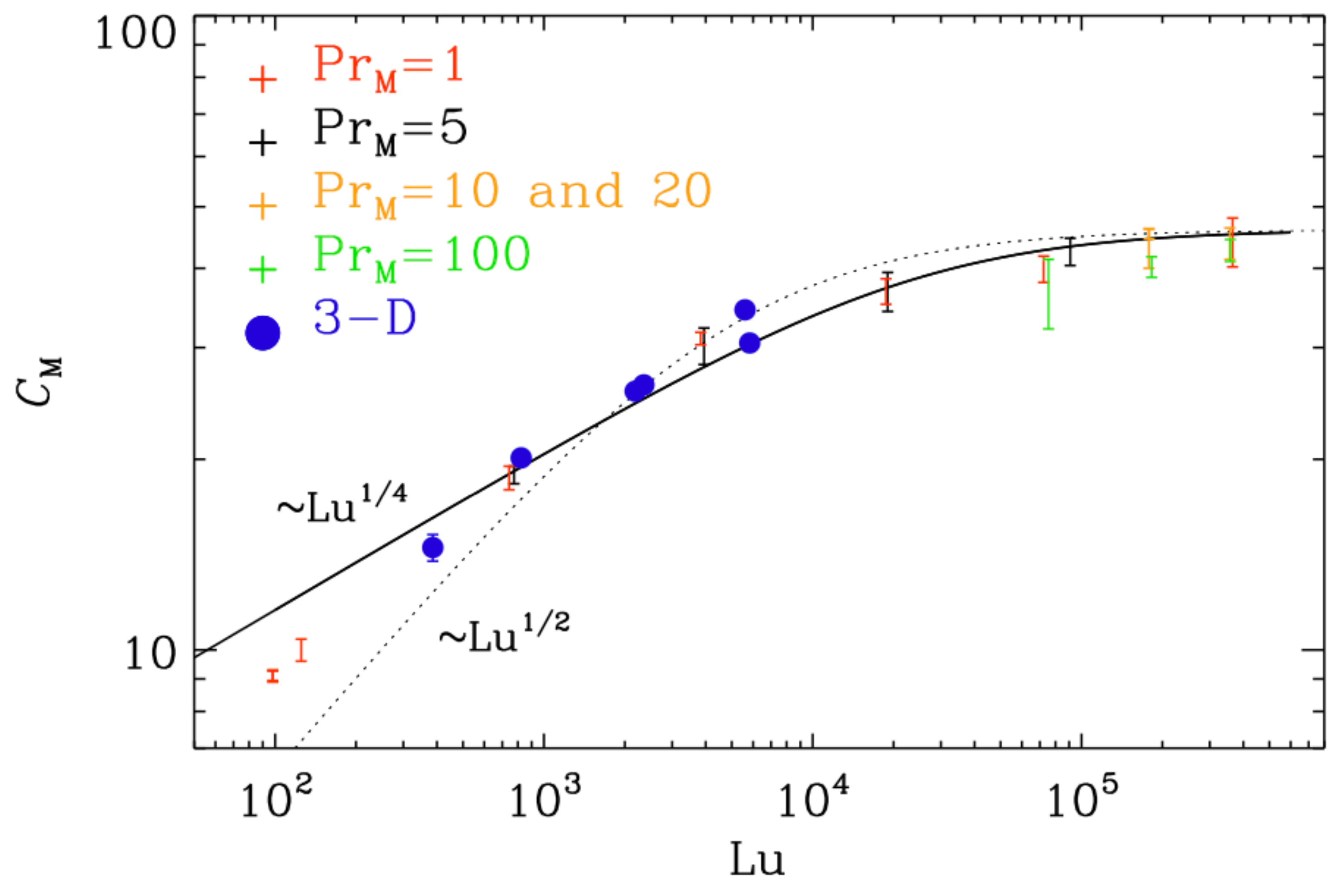
The reconnection rate is then (Uzdensky et. al. 2010)

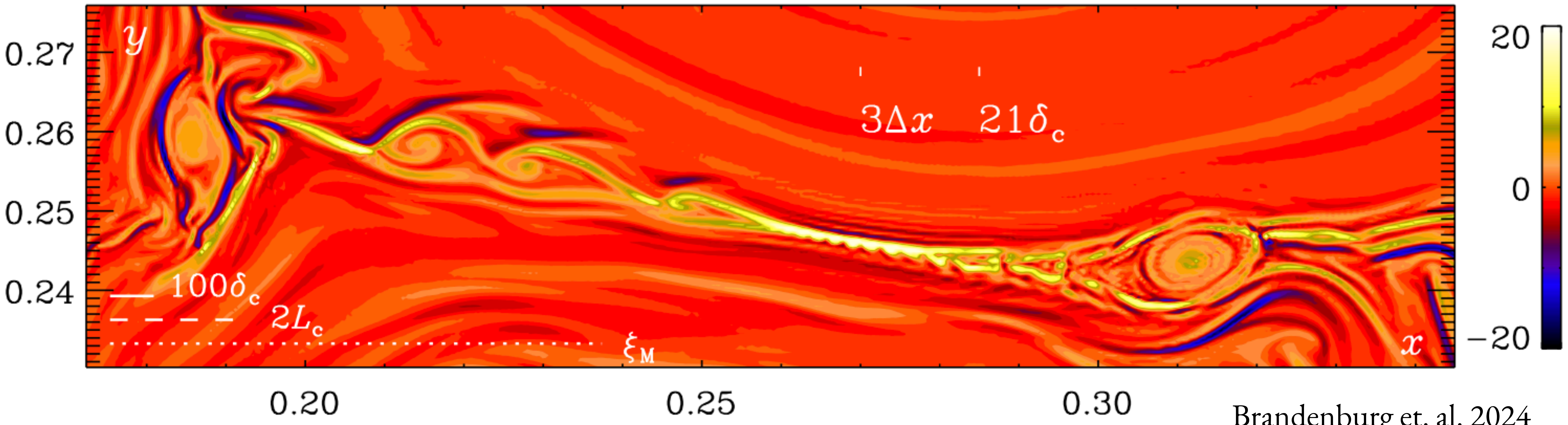
$$\frac{1}{\tau_{\rm rec}} \equiv \frac{u_{\rm in}}{\lambda} \sim \frac{1}{S_c^{1/2}\sqrt{1+{\rm Pm}}}\frac{v_A}{\lambda}$$

at recombination: $S_c \sim 10^4, \quad {\rm Pm} \sim 10^7 \implies \frac{1}{\tau_{\rm rec}} \sim 10^{-5.5}\frac{v_A}{\lambda}$

Numerical evidence for plasmoid-mediated decay laws

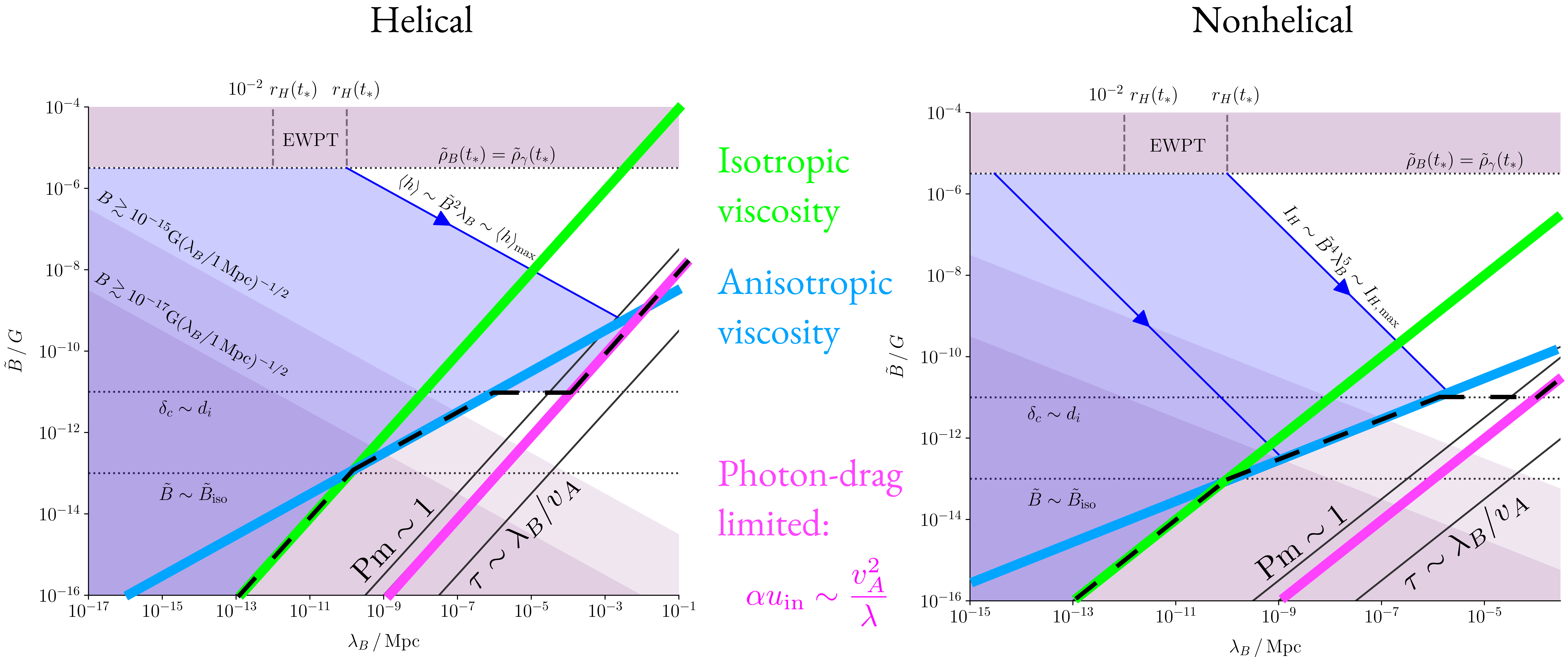


- ✅ Dependence on Lundquist number S
- ? Dependence on Pm



Brandenburg et. al. 2024

Reconnection-controlled decay of PMFs



Hosking & Schekochihin 2023

Conclusions

The decay of primordial magnetic fields is likely controlled by topological invariants related to magnetic helicity, whose relevance to decay is precisely that they are conserved *even during magnetic reconnection.*

Reconnection allows the decaying fields to access lower-energy states (Taylor relaxation).

Thus, we expect the decay timescale to be the one for magnetic reconnection. This results in significant suppression of the decay, owing to the large magnetic Prandtl number of the early Universe.