Gravitational Waves from Chiral Plasma Instability in the Standard Cosmology

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May 3, 2024

Baryogenesis "by-products"

Among the outstanding problems in modern cosmology (dark matter, dark energy, inflation, baryogenesis) … the matter / anti-matter asymmetry is uniquely challenging, because we only know one number ($n_B/s = 10^{-10}$)!

Therefore it is compelling to study models with "secondary predictions" that we can test in the lab (*e.g.*, EWBG tested by collider observables & EDMs).

However, the physics of baryogenesis may not within reach of terrestrial experiments (*e.g.*, Affleck-Dine, GUT baryogenesis, high-scale leptogenesis).

In this case, we may still probe the origin of the matter / anti-matter asymmetry through observations of baryogenesis "by-products".

Baryogenesis requires a departure from thermal equilibrium (Sakharov), and such conditions may create additional cosmological relics (*e.g.*, gravity waves and topological defects) or the OOE conditions may be provided by other relics (*e.g.*, primordial black holes and primordial magnetic fields).

If we could observe these other relics (spectra), we would gain a new handle on the origin of the matter / anti-matter asymmetry (more numbers) .

PHYSICAL REVIEW D **109,** 043534 (2024)

Relic gravitational waves from the chiral plasma instability in the standard cosmological model

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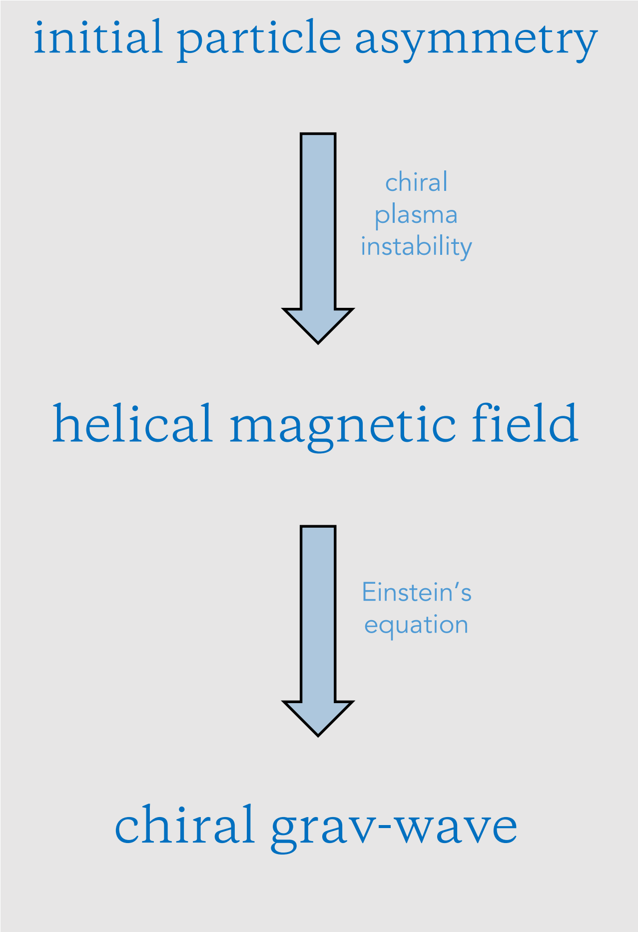
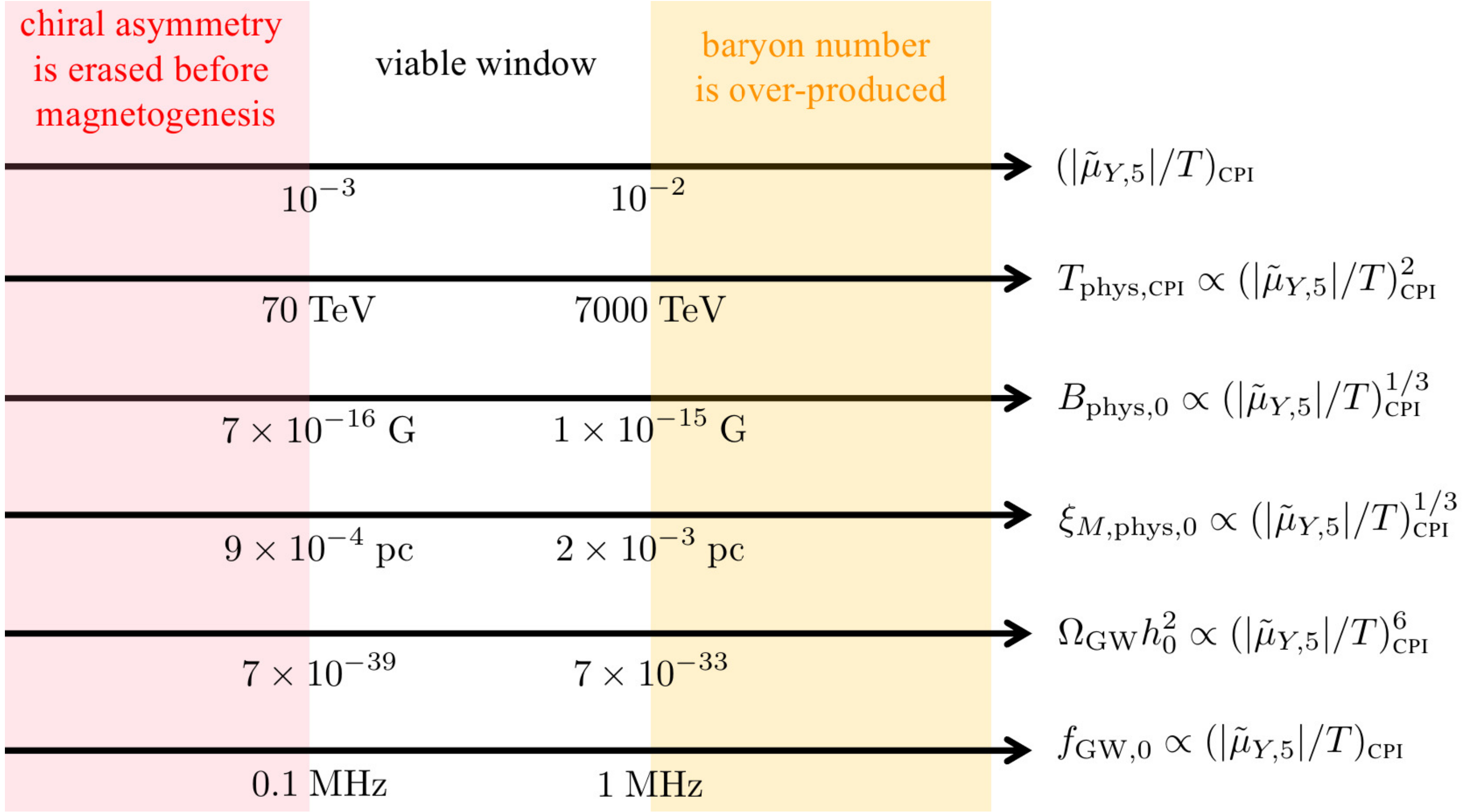
(Received 19 July 2023; accepted 25 January 2024; published 27 February 2024)

In the primordial plasma, at temperatures above the scale of electroweak symmetry breaking, the presence of chiral asymmetries is expected to induce the development of helical hypermagnetic fields through the phenomenon of chiral plasma instability. It results in magnetohydrodynamic turbulence due to the high conductivity and low viscosity and sources gravitational waves that survive in the universe today as a stochastic polarized gravitational wave background. In this article, we show that this scenario only relies on Standard Model physics, and therefore the observable signatures, namely the relic magnetic field and gravitational background, are linked to a single parameter controlling the initial chiral asymmetry. We estimate the magnetic field and gravitational wave spectra, and validate these estimates with 3D numerical simulations.

DOI: 10.1103/PhysRevD.109.043534

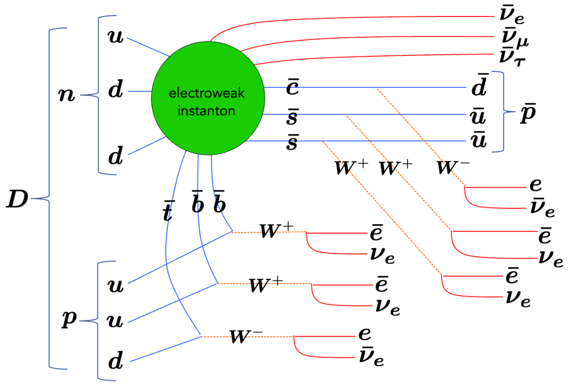


where I'm going with this:



if there's time at the end:

Standard Model deuteron decay



the electroweak plasma
& particle asymmetries

The electroweak plasma

We're thinking about the primordial plasma before the electroweak epoch.

FRW spacetime (homo / iso)

$$(\mathrm{d}s)^2 = a(t)^2 \left[(\mathrm{d}t)^2 - |\mathrm{d}\boldsymbol{x}|^2 \right]$$

$$H(t) = \frac{1}{a(t)} \frac{\mathrm{d}}{\mathrm{d}t} a(t)$$

R-dominated

$$\mathcal{E}(t) = \frac{\pi^2}{30} g_{*E}(t)\, T(t)^4$$

comoving entropy constant

$$s(t) = \frac{2\pi^2}{45} g_{*S}(t) T(t)^3 = \frac{2\pi^2}{45} g_{*S,0} T_0^3$$

at the electroweak epoch:

$$T_{\mathrm{phys,ew}} \simeq 100\ \mathrm{GeV}$$

$$t_{\mathrm{phys,ew}} \simeq (2.3 \times 10^{-11}\ \mathrm{sec}) \left(\frac{g_*}{106.75} \right)^{-1/2}$$

$$a_{\mathrm{ew}} \simeq (7.8 \times 10^{-16}\ a_0) \left(\frac{g_*}{106.75} \right)^{-1/3}$$

$$d_{H,\mathrm{phys,ew}} \simeq (1.4\ \mathrm{cm}) \left(\frac{g_*}{106.75} \right)^{-1/2}$$

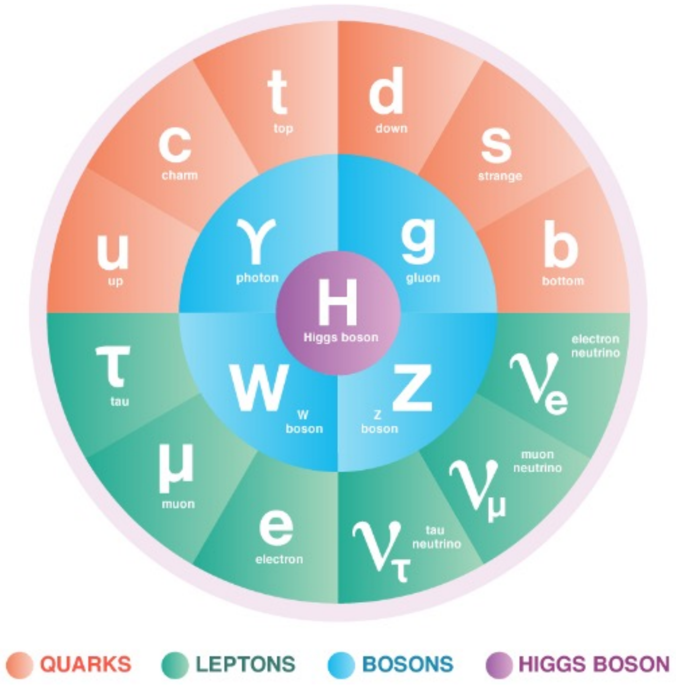
A note on notation

All dimensionful variables are comoving. Some examples,

conformal time:	$\mathrm{d}t = \mathrm{d}t_{\mathrm{phys}}/a$
comoving Hubble param:	$H = aH_{\mathrm{phys}}$
comoving temperature:	$T = aT_{\mathrm{phys}}$
comoving energy density:	$\mathcal{E} = a^4 \mathcal{E}_{\mathrm{phys}}$
comoving chemical potential:	$\mu = a\mu_{\mathrm{phys}}$
comoving magnetic field strength:	$\boldsymbol{B} = a^2 \boldsymbol{B}_{\mathrm{phys}}$
comoving magnetic correlation length:	$\xi_M = \xi_{M,\mathrm{phys}}/a$
comoving conductivity:	$\sigma = a\sigma_{\mathrm{phys}}$
Friedmann equation:	$3a^2 M_{\mathrm{pl}}^2 H^2 = \mathcal{E}$

Particle content of the universe

The Standard Model of the Elementary Particles



electroweak unbroken phase

$$\langle \Phi \rangle = 0$$
$$\text{mass}(W^{\pm}, Z) = 0$$
$$\text{mass}(\text{quarks, leptons}) = 0$$

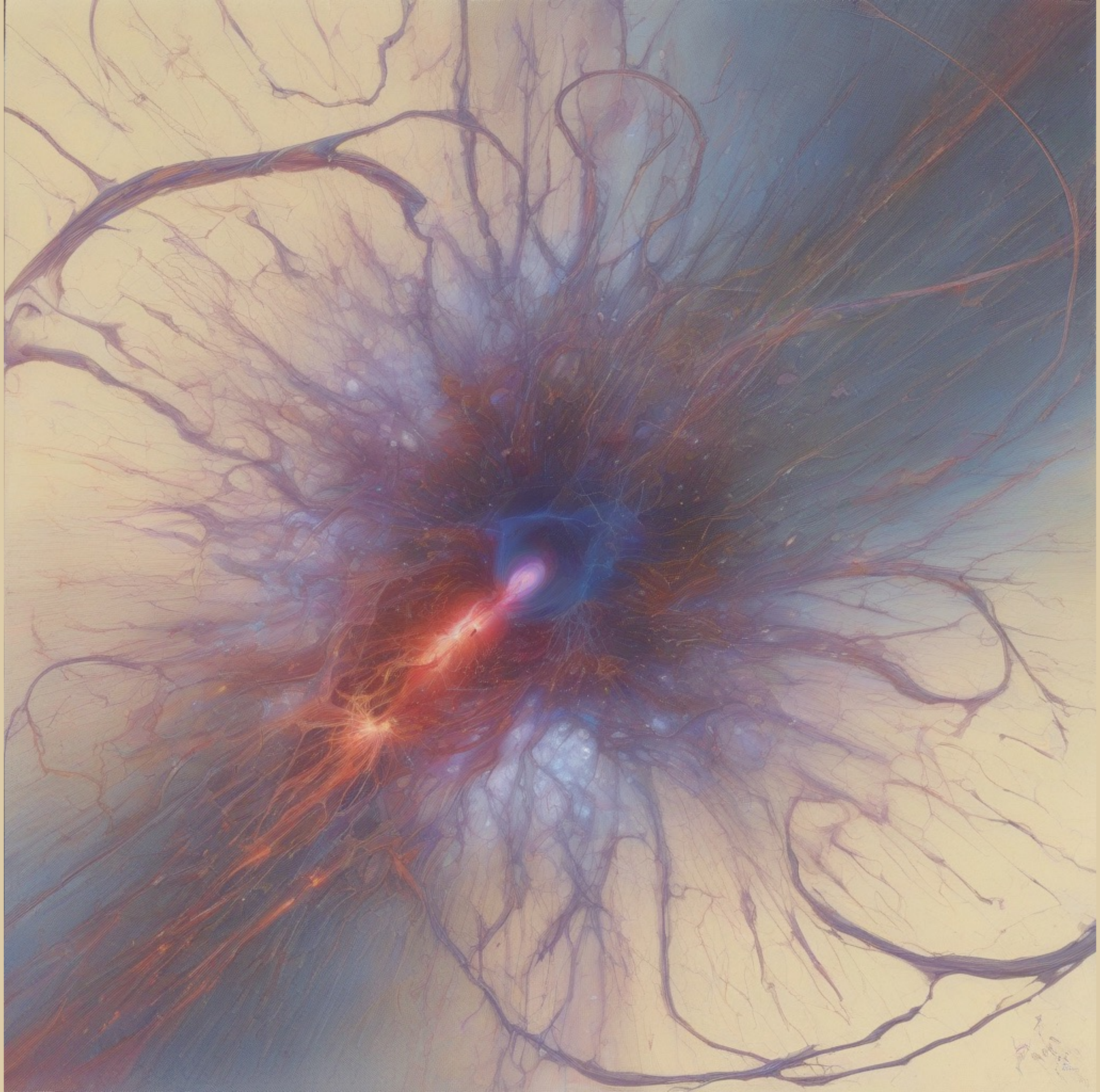
gauge theory

$$\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$$

field content

$$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \sim (\mathbf{3}\,,\, \mathbf{2}\,,\, 1/3)$$
$$u_R \sim (\mathbf{3}\,,\, \mathbf{1}\,,\, 4/3)$$
$$d_R \sim (\mathbf{3}\,,\, \mathbf{1}\,,\, -2/3)$$
$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim (\mathbf{1}\,,\, \mathbf{2}\,,\, -1)$$
$$e_R \sim (\mathbf{1}\,,\, \mathbf{1}\,,\, -2)$$
$$G \sim (\mathbf{8}\,,\, \mathbf{1}\,,\, 0)$$
$$W \sim (\mathbf{1}\,,\, \mathbf{3}\,,\, 0)$$
$$B \sim (\mathbf{1}\,,\, \mathbf{1}\,,\, 0)$$
$$\Phi \sim (\mathbf{1}\,,\, \mathbf{2}\,,\, 1)$$

AI Image Generator
“primordial
electroweak plasma”



Particle asymmetries

All of the SM particles interact (scatter, decay, inverse-decay) & maintain thermal equilibrium

momentum distribution

$$\mathrm{d}n_i(t) = g_i \, \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3} \, f_i(\boldsymbol{p}, t)$$

$$f_i(\boldsymbol{p}, t) = \begin{cases} \left(\mathrm{e}^{[E_i(t)-\mu_i(t)]/T(t)} - 1\right)^{-1} \\ \left(\mathrm{e}^{[E_i(t)-\mu_i(t)]/T(t)} + 1\right)^{-1} \end{cases}$$

$$E_i(t) = \sqrt{|\boldsymbol{p}|^2 + a(t)^2 m_i^2}$$

number density (for small μ_i)

$$n_i(t) = g_i \begin{cases} \frac{\zeta(3)}{\pi^2} T^3 + \frac{1}{6}\mu_i T^2 + O(\mu_i^2 T) \\ \frac{3}{4}\frac{\zeta(3)}{\pi^2} T^3 + \frac{1}{12}\mu_i T^2 + O(\mu_i^2 T) \end{cases}$$

if pair annihilation & production are in thermal equilibrium:

$$\mu_i = -\mu_{\bar{i}}$$

particle asymmetries

$$\Delta n_i(t) = g_i \begin{cases} \frac{1}{3}\mu_i T^2 + O(\mu_i^2 T) \\ \frac{1}{6}\mu_i T^2 + O(\mu_i^2 T) \end{cases}$$

Standard Model Boltzmann Equations w/ Anomalous Sources

$$
\begin{aligned}
\frac{d\eta_{u_L^i}}{dx} &= -\mathcal{S}^{i}_{\rm UDW} - \sum_{j=1}^{N_{\rm g}} \left(\mathcal{S}^{ij}_{\rm Uhu} + \mathcal{S}^{ij}_{\rm Uu} + \mathcal{S}^{ij}_{\rm Uhd} \right) - \mathcal{S}_{\rm s,sph} - \frac{N_{\rm c}}{2} \mathcal{S}_{\rm w,sph} \\
&\quad + \left(N_{\rm c} y_{Q_L}^2 \mathcal{S}^{\rm bkg}_{\rm y} + \frac{N_{\rm c}}{2} \mathcal{S}^{\rm bkg}_{\rm w} + N_{\rm c} \frac{y_{Q_L}}{2} \mathcal{S}^{\rm bkg}_{\rm yw} \right) \\
\frac{d\eta_{d_L^i}}{dx} &= \mathcal{S}^{i}_{\rm UDW} - \sum_{j=1}^{N_{\rm g}} \left(\mathcal{S}^{ij}_{\rm Dhd} + \mathcal{S}^{ij}_{\rm Dd} + \mathcal{S}^{ij}_{\rm Dhu} \right) - \mathcal{S}_{\rm s,sph} - \frac{N_{\rm c}}{2} \mathcal{S}_{\rm w,sph} \\
&\quad + \left(N_{\rm c} y_{Q_L}^2 \mathcal{S}^{\rm bkg}_{\rm y} + \frac{N_{\rm c}}{2} \mathcal{S}^{\rm bkg}_{\rm w} - N_{\rm c} \frac{y_{Q_L}}{2} \mathcal{S}^{\rm bkg}_{\rm yw} \right) \\
\frac{d\eta_{\nu_L^i}}{dx} &= -\mathcal{S}^{i}_{\nu\rm EW} - \sum_{j=1}^{N_{\rm g}} \mathcal{S}^{ij}_{\nu\rm he} - \frac{1}{2} \mathcal{S}_{\rm w,sph} + \left(y_{L_L}^2 \mathcal{S}^{\rm bkg}_{\rm y} + \frac{1}{2} \mathcal{S}^{\rm bkg}_{\rm w} + \frac{y_{L_L}}{2} \mathcal{S}^{\rm bkg}_{\rm yw} \right) \\
\frac{d\eta_{e_L^i}}{dx} &= \mathcal{S}^{i}_{\nu\rm EW} - \sum_{j=1}^{N_{\rm g}} \left(\mathcal{S}^{ij}_{\rm Ehe} + \mathcal{S}^{ij}_{\rm Ee} \right) - \frac{1}{2} \mathcal{S}_{\rm w,sph} + \left(y_{L_L}^2 \mathcal{S}^{\rm bkg}_{\rm y} + \frac{1}{2} \mathcal{S}^{\rm bkg}_{\rm w} - \frac{y_{L_L}}{2} \mathcal{S}^{\rm bkg}_{\rm yw} \right) \\
\frac{d\eta_{u_R^i}}{dx} &= \sum_{j=1}^{N_{\rm g}} \left(\mathcal{S}^{ji}_{\rm Uhu} + \mathcal{S}^{ji}_{\rm Uu} + \mathcal{S}^{ji}_{\rm Dhu} \right) + \mathcal{S}_{\rm s,sph} - N_{\rm c} y_{u_R}^2 \mathcal{S}^{\rm bkg}_{\rm y} \\
\frac{d\eta_{d_R^i}}{dx} &= \sum_{j=1}^{N_{\rm g}} \left(\mathcal{S}^{ji}_{\rm Dhd} + \mathcal{S}^{ji}_{\rm Dd} + \mathcal{S}^{ji}_{\rm Uhd} \right) + \mathcal{S}_{\rm s,sph} - N_{\rm c} y_{d_R}^2 \mathcal{S}^{\rm bkg}_{\rm y} \\
\frac{d\eta_{e_R^i}}{dx} &= \sum_{j=1}^{N_{\rm g}} \left(\mathcal{S}^{ji}_{\rm Ehe} + \mathcal{S}^{ji}_{\rm Ee} + \mathcal{S}^{ji}_{\nu\rm he} \right) - y_{e_R}^2 \mathcal{S}^{\rm bkg}_{\rm y} \\
\frac{d\eta_{\phi^+}}{dx} &= -\left(\mathcal{S}_{\rm hhw} + \mathcal{S}_{\rm hw} \right) + \sum_{i,j=1}^{N_{\rm g}} \left(-\mathcal{S}^{ij}_{\rm Dhu} + \mathcal{S}^{ij}_{\rm Uhd} + \mathcal{S}^{ij}_{\nu\rm he} \right) \\
\frac{d\eta_{\phi^0}}{dx} &= \mathcal{S}_{\rm hhw} - \mathcal{S}_{\rm h} + \sum_{i,j=1}^{N_{\rm g}} \left(-\mathcal{S}^{ij}_{\rm Uhu} + \mathcal{S}^{ij}_{\rm Dhd} + \mathcal{S}^{ij}_{\rm Ehe} \right) \\
\frac{d\eta_{W^+}}{dx} &= \left(\mathcal{S}_{\rm hhw} + \mathcal{S}_{\rm hw} \right) + \sum_{i=1}^{N_{\rm g}} \left(\mathcal{S}^{i}_{\rm UDW} + \mathcal{S}^{i}_{\nu\rm EW} \right) .
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{S}^{ij}_{\rm Dhu} &\equiv \frac{\gamma^{ij}_{\rm Dhu}}{2} \left(\frac{\eta_{d_L^i}}{k_{d_L^i}} + \frac{\eta_{\phi^+}}{k_{\phi^+}} - \frac{\eta_{u_R^j}}{k_{u_R^j}} \right) , \quad
\mathcal{S}^{ij}_{\rm Uhu} \equiv \frac{\gamma^{ij}_{\rm Uhu}}{2} \left(\frac{\eta_{u_L^i}}{k_{u_L^i}} + \frac{\eta_{\phi^0}}{k_{\phi^0}} - \frac{\eta_{u_R^j}}{k_{u_R^j}} \right) , \\
\mathcal{S}^{ij}_{\rm Uhd} &\equiv \frac{\gamma^{ij}_{\rm Uhd}}{2} \left(\frac{\eta_{u_L^i}}{k_{u_L^i}} - \frac{\eta_{\phi^+}}{k_{\phi^+}} - \frac{\eta_{d_R^j}}{k_{d_R^j}} \right) , \quad
\mathcal{S}^{ij}_{\rm Dhd} \equiv \frac{\gamma^{ij}_{\rm Dhd}}{2} \left(\frac{\eta_{d_L^i}}{k_{d_L^i}} - \frac{\eta_{\phi^0}}{k_{\phi^0}} - \frac{\eta_{d_R^j}}{k_{d_R^j}} \right) , \\
\mathcal{S}^{ij}_{\nu\rm he} &\equiv \frac{\gamma^{ij}_{\nu\rm he}}{2} \left(\frac{\eta_{\nu_L^i}}{k_{\nu_L^i}} - \frac{\eta_{\phi^+}}{k_{\phi^+}} - \frac{\eta_{e_R^j}}{k_{e_R^j}} \right) , \quad
\mathcal{S}^{ij}_{\rm Ehe} \equiv \frac{\gamma^{ij}_{\rm Ehe}}{2} \left(\frac{\eta_{e_L^i}}{k_{e_L^i}} - \frac{\eta_{\phi^0}}{k_{\phi^0}} - \frac{\eta_{e_R^j}}{k_{e_R^j}} \right) ,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{S}^{i}_{\rm UDW} &\equiv \gamma^{i}_{\rm UDW} \left(\frac{\eta_{u_L^i}}{k_{u_L^i}} - \frac{\eta_{d_L^i}}{k_{d_L^i}} - \frac{\eta_{W^+}}{k_{W^+}} \right) \\
\mathcal{S}^{i}_{\nu\rm EW} &\equiv \gamma^{i}_{\nu\rm EW} \left(\frac{\eta_{\nu_L^i}}{k_{\nu_L^i}} - \frac{\eta_{e_L^i}}{k_{e_L^i}} - \frac{\eta_{W^+}}{k_{W^+}} \right) \\
\mathcal{S}_{\rm hhw} &\equiv \gamma_{\rm hhw} \left(\frac{\eta_{\phi^+}}{k_{\phi^+}} - \frac{\eta_{\phi^0}}{k_{\phi^0}} - \frac{\eta_{W^+}}{k_{W^+}} \right)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{S}_{\rm s,sph} &\equiv \gamma_{\rm s,sph} \sum_{i=1}^{N_{\rm g}} \left(\frac{\eta_{u_L^i}}{k_{u_L^i}} + \frac{\eta_{d_L^i}}{k_{d_L^i}} - \frac{\eta_{u_R^i}}{k_{u_R^i}} - \frac{\eta_{d_R^i}}{k_{d_R^i}} \right) , \\
\mathcal{S}_{\rm w,sph} &\equiv \gamma_{\rm w,sph} \sum_{i=1}^{N_{\rm g}} \left(\frac{N_{\rm c}}{2} \frac{\eta_{u_L^i}}{k_{u_L^i}} + \frac{N_{\rm c}}{2} \frac{\eta_{d_L^i}}{k_{d_L^i}} + \frac{1}{2} \frac{\eta_{\nu_L^i}}{k_{\nu_L^i}} + \frac{1}{2} \frac{\eta_{e_L^i}}{k_{e_L^i}} \right)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{S}^{ij}_{\rm Uu} &\equiv \gamma^{ij}_{\rm Uu} \left(\frac{\eta_{u_L^i}}{k_{u_L^i}} - \frac{\eta_{u_R^j}}{k_{u_R^j}} \right) , \\
\mathcal{S}^{ij}_{\rm Dd} &\equiv \gamma^{ij}_{\rm Dd} \left(\frac{\eta_{d_L^i}}{k_{d_L^i}} - \frac{\eta_{d_R^j}}{k_{d_R^j}} \right) , \\
\mathcal{S}^{ij}_{\rm Ee} &\equiv \gamma^{ij}_{\rm Ee} \left(\frac{\eta_{e_L^i}}{k_{e_L^i}} - \frac{\eta_{e_R^j}}{k_{e_R^j}} \right) , \\
\mathcal{S}_{\rm hw} &\equiv \gamma_{\rm hw} \left(\frac{\eta_{\phi^+}}{k_{\phi^+}} - \frac{\eta_{W^+}}{k_{W^+}} \right) \\
\mathcal{S}_{\rm h} &\equiv \gamma_{\rm h} \frac{\eta_{\phi^0}}{k_{\phi^0}} .
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{S}^{\rm bkg}_{\rm y} &= \frac{1}{sT} \frac{\alpha_{\rm y}}{4\pi} \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \overline{\langle Y_{\mu\nu} \rangle \langle Y_{\rho\sigma} \rangle} \\
\mathcal{S}^{\rm bkg}_{\rm w} &= \frac{1}{sT} \frac{1}{2} \frac{\alpha_{\rm w}}{4\pi} \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \overline{\langle W^a_{\mu\nu} \rangle \langle W^a_{\rho\sigma} \rangle} \\
\mathcal{S}^{\rm bkg}_{\rm yw} &= \frac{1}{sT} \frac{gg'/4\pi}{4\pi} \epsilon^{\mu\nu\rho\sigma} \overline{\langle Y_{\mu\nu} \rangle \langle W^3_{\rho\sigma} \rangle} .
\end{aligned}
$$

$\eta = n/s$

$x = T/H \sim M_{\rm pl}/T$

$k = \#$ degree of freedom

Hypercharge-weighted chiral asymmetry

Some particle asymmetries correspond to (quasi-)conserved charges. They're important.

For example, baryon-number

$$\mathsf{B}_i = \begin{cases} +1/3 & \text{for quarks} \\ -1/3 & \text{for anti-quarks} \\ 0 & \text{otherwise} \end{cases}$$

$$n_{\mathsf{B}}(t) = \sum_i \mathsf{B}_i n_i \approx \tfrac{1}{6}\mu_{\mathsf{B}} T^2 \quad \text{where} \quad \mu_{\mathsf{B}} = \sum_{\text{particles}} g_i \mathsf{B}_i \mu_i$$

We're particularly interested in the hypercharge-weighted chiral asymmetry:

$$\mu_{Y,5}(t) = \sum_{\text{particles}} g_i \varepsilon_i Y_i^2 \mu_i(t)$$

$\varepsilon_i = \pm 1$ for R- and L-chiral particles

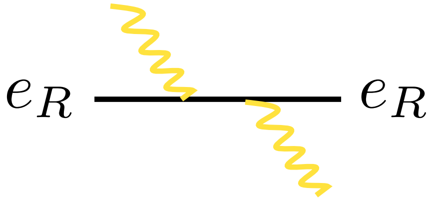
$Y_i =$ hypercharge of particle species i

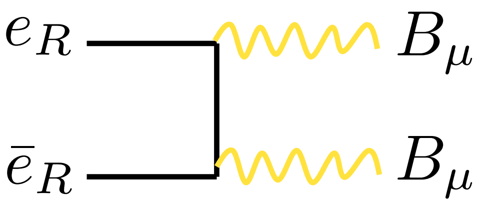
particle	ε_i	Y_i	$\varepsilon_i Y_i^2$
Q	-1	$1/3$	$-1/9$
u_R	$+1$	$4/3$	$16/9$
d_R	$+1$	$-2/3$	$4/9$
L	-1	-1	-1
e_R	$+1$	-2	4
G, W, B, Φ	0	$\cdot$	0

Interactions & charge conservation

Some charges are exactly conserved. The corresponding particle asymmetries are static.

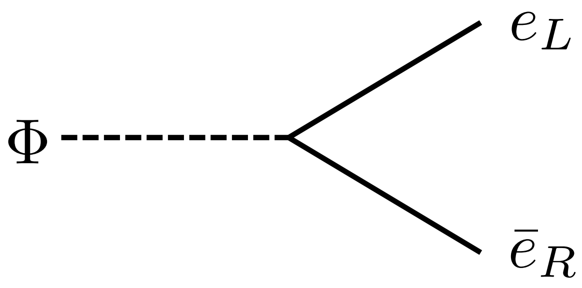
e.g., hypercharge … no interaction can change this





But some charges are only approximately conserved (on short time scales).

e.g., the Yukawa interactions change chiral charge



	hypercharge	Y-chiral
for Φ:	$Y = +1$	$\varepsilon Y^2 = 0$
for e_L:	$Y = -1$	$\varepsilon Y^2 = -1$
for $\bar{e}_R$:	$Y = +2$	$\varepsilon Y^2 = -4$
	(conserved)	(violated)

Chiral charge erasure

rate comes from: Bodeker & Schroder (2019)
see also: Cambbell, Davidson, Ellis, & Olive (1993)
Joyce & Shaposhnikov (1997)

The approximately conserved charges tend to be erased over time.

The limiting factor is the right-chiral electron, since it has $y_e \sim 3 * 10^{-6}$.

chiral charge evolution

$$\frac{\mathrm{d}}{\mathrm{d}t}\mu_{Y,5}(t) = -\Gamma_{\mathrm{f}}(t)\,\mu_{Y,5}(t)$$

interaction rate

$$\Gamma_{\mathrm{f}}(t) \sim \left(1.3 \times 10^{-2}\right) y_e^2 T(t)$$

an exponential decay:

$$\mu_{Y,5}(t) = \mu_{Y,5,0}\,\exp\left[-\int_{t_0}^{t} \mathrm{d}t'\,\Gamma_f(t')\right]$$

chiral-erasure epoch

$$T_{\mathrm{phys,f}} \simeq (80\ \mathrm{TeV}) \left(\frac{g_*}{106.75}\right)^{-1/2}$$

$$t_{\mathrm{phys,f}} \simeq (3.7 \times 10^{-17}\ \mathrm{sec}) \left(\frac{g_*}{106.75}\right)^{1/2}$$

$$a_{\mathrm{f}} \simeq (9.7 \times 10^{-19}\ a_0) \left(\frac{g_*}{106.75}\right)^{1/6}$$

$$d_{H,\mathrm{phys,f}} \simeq (2.2 \times 10^{-6}\ \mathrm{cm}) \left(\frac{g_*}{106.75}\right)^{1/2}$$

key message thus far

There might be a primordial chiral asymmetry (e.g., $e_R > ebar_R$).

Yukawa interactions will erase the asymmetry once the plasma temperature drops to ~ 80 TeV.

If we want to do something fun with the chiral asymmetry, we need to finish before T = 80 TeV (or continue to source the asymmetry later, cf. Misha's talk)

magnetic field evolution
& chiral magnetic effect

A hypermagnetic field

mag screening in QCD: Gross, Pisarski, & Yaffe (1981)
see also: Arnold, Son, & Yaffe (1996)

Where can we add magnetic fields to our primordial plasma?
(for T > 100 GeV electroweak unbroken phase)

(1) electromagnetism is in here: $A_\mu = \cos\theta_W \, B_\mu + \sin\theta_W \, W_\mu^3$

gauge interactions: $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$

(2) but non-Abelian magnetic fields are screened at finite temperature

$$m_{\mathrm{mag}} \sim g^2 T$$

(3) so the only place for long-range coherent magnetic fields is here

$$\boldsymbol{B}_Y(\eta, \boldsymbol{x}) \quad \text{and} \quad \boldsymbol{E}_Y(\eta, \boldsymbol{x})$$

Evolution of hypermagnetic fields

conductivity: Arnold, Moore, & Yaffe (2000)

How do hypermagnetic fields evolve in the primordial plasma?

hyper-Maxwell equations

$$\boldsymbol{\nabla} \cdot \boldsymbol{E}_Y = \rho_Y$$

$$\boldsymbol{\nabla} \times \boldsymbol{E}_Y = -\frac{\partial}{\partial t} \boldsymbol{B}_Y$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B}_Y = 0$$

$$\boldsymbol{\nabla} \times \boldsymbol{B}_Y = \boldsymbol{J}_Y + \frac{\partial}{\partial t} \boldsymbol{E}_Y$$

continuity equation

$$\frac{\partial}{\partial \eta} \rho_Y + \boldsymbol{\nabla} \cdot \boldsymbol{J}_Y = 0$$

constitutive relation

$$\boldsymbol{J}_Y = \underbrace{\sigma_Y \left(\boldsymbol{E}_Y + \boldsymbol{v} \times \boldsymbol{B}_Y \right)}_{\text{Ohm's Law}} + \underbrace{\frac{2}{\pi} \alpha_Y \mu_{Y,5} \boldsymbol{B}_Y}_{\text{chiral magnetic effect (CME)}}$$

fine structure constant

$$\alpha_Y = g'^2 / 4\pi \approx 1/100$$

conductivity

$$\sigma_Y \sim T/\alpha_Y \approx 100T$$

magnetic diffusivity

$$\eta_Y = 1/\sigma_Y \approx 0.01 T^{-1}$$

Chiral magnetic effect

Vilenkin (1980), Fukushima, Kharzeev, & Warringa (2008),
Alekseev, Cheianov, & Frohlich (1998)
Boyarsky, Frohlich, & Ruchaiskiy (2011)

First studied for a relativistic electron-positron plasma in QED.

CME = in the presence of a chiral asymmetry, a magnetic field induces an electric current.

How can we understand this effect?

consider, is this allowed: $\boldsymbol{J} = m_e \, \boldsymbol{B}$ (?)

mass dimensions: ✓

parity properties: ✗

On the other hand, the CME has:

$$\boldsymbol{J} \propto \mu_5 \boldsymbol{B}$$

mass dimensions: ✓

parity properties: ✓

$\boldsymbol{J}$ = vector = P-odd
$\boldsymbol{B}$ = axial-vector = P-even
m_e = scalar = P-even
μ_5 = pseudo-scalar = P-odd
$\mu_5 \propto [\#\text{ R-chiral}] - [\#\text{ L-chiral}]$

From the symmetry perspective, the CME could be correct.

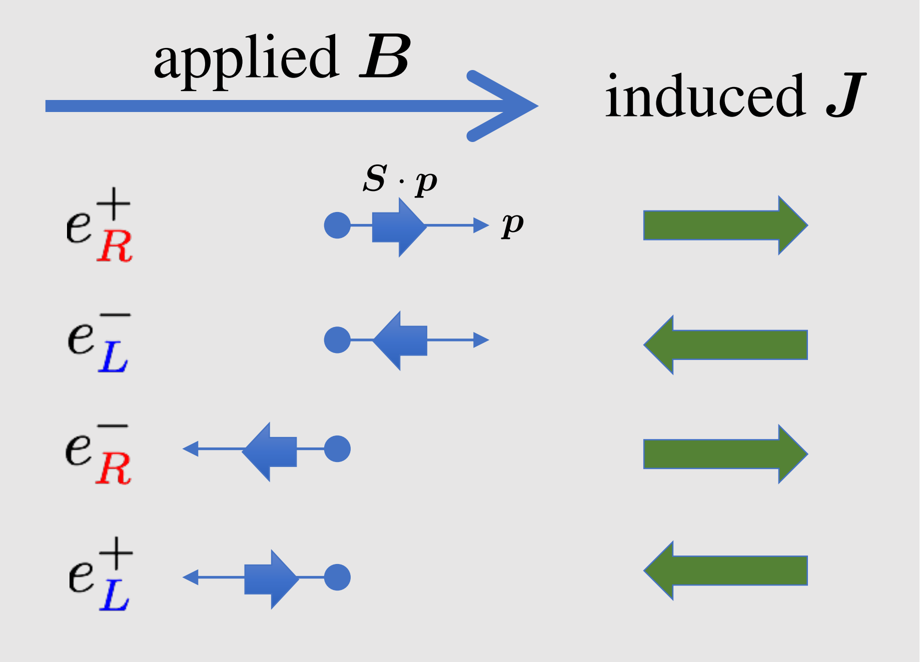
Semiclassical understanding

Tashiro, Vachaspati, & Vilenkin (2012)

Consider, applying a B-field to a collection of (relativistic) electrons & positrons.

The magnetic dipole moment **μ** ~ qS wants to align with B … the Hamiltonian is: $H = -\boldsymbol{\mu} \cdot \boldsymbol{B}$

without chiral asymmetry

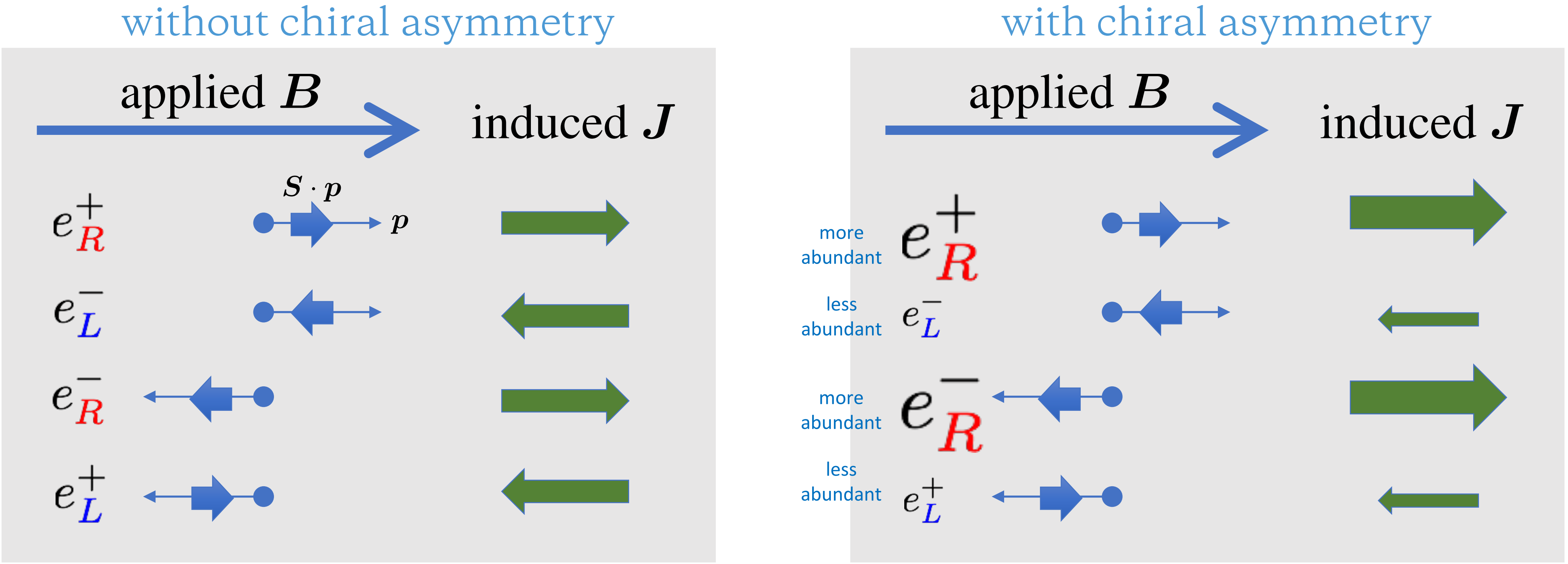


Heuristic understanding

Tashiro, Vachaspati, & Vilenkin (2012)

Consider, applying a B-field to a collection of relativistic electrons & positrons.

The magnetic dipole moment **μ** ~ q**S** wants to align with **B** … the Hamiltonian is: $H = -\boldsymbol{\mu} \cdot \boldsymbol{B}$



implies: $\boldsymbol{J} \propto q^2 \mu_5 \boldsymbol{B}$

CME recap

CME = in the presence of a chiral asymmetry, a magnetic field induces an electric current.

for electromagnetism: $\boldsymbol{J} = \frac{2}{\pi} \alpha \mu_5 \boldsymbol{B}$

for hypercharge:

$$\boldsymbol{J}_Y = \frac{2}{\pi} \alpha_Y \mu_{Y,5} \boldsymbol{B}_Y$$

$$\mu_{Y,5}(t) = \sum_{\text{particles}} g_i \varepsilon_i Y_i^2 \mu_i(t)$$

$$\varepsilon_i = \pm 1 \text{ for R- and L-chiral particles}$$

$$Y_i = \text{hypercharge of particle species } i$$

magnetogenesis
via chiral plasma instability

Linearized field equations

similar equations of motion to axion inflation
(cf, talks by Lorenzo & Rajeev)

Combining the hyper-Maxwell equations (and neglecting velocity v)

$$0 = \dot{\boldsymbol{B}}_Y - \eta_Y \nabla^2 \boldsymbol{B}_Y - \tfrac{2}{\pi} \alpha_Y \eta_Y \mu_{Y,5} \boldsymbol{\nabla} \times \boldsymbol{B}_Y$$

Move to Fourier space (and assume homogenous μ_{Y5}).

Circular polarization modes decouple

$$0 = \dot{B}^{(R)}_{Y,k} + \eta_Y \left(k^2 - \mathrm{sgn}(\mu_{Y,5})\, k_{\mathrm{CPI}} k \right) B^{(R)}_{Y,k}$$

$$0 = \dot{B}^{(L)}_{Y,k} + \eta_Y \left(k^2 + \mathrm{sgn}(\mu_{Y,5})\, k_{\mathrm{CPI}} k \right) B^{(L)}_{Y,k}$$

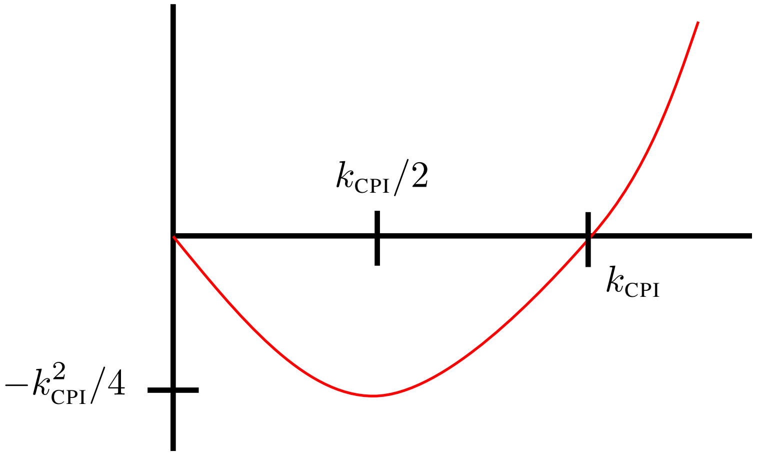
where $k_{\mathrm{CPI}} = \tfrac{2}{\pi} \alpha_Y |\mu_{Y,5}|$

Depending on the sign of μ_{Y5}, one mode or the other can be unstable.

Chiral plasma instability

Explore the dependence on wavenumber

$$0 = \dot{B}^{(R)}_{Y,k} + \eta_Y \left(k^2 - k_{\rm CPI} k \right) B^{(R)}_{Y,k}$$



- modes with $k > k_{CPI}$ are decaying (Ohmic dissipation)
- modes with $k < k_{CPI}$ have a tachyonic instability
- modes with $k = k_{CPI}/2$ grow fastest

$$B^{(R)}_{Y,k}(t) \propto \exp\left[\tfrac{1}{4}\eta_Y k^2_{\rm CPI} t\right]$$

- typically k_{CPI} is deep inside the horizon

CPI = tachyonic growth of a helical magnetic field due to the CME

Helicity saturation

Since either the R- or L-polarized modes grow, the resultant magnetic field is helical.

As the helicity grows, the chiral asymmetry is depleted:

$$\partial_\mu j^\mu_{Y,5} \sim -\frac{g'^2}{16\pi^2} Y_{\mu\nu}\tilde{Y}^{\mu\nu} \quad \Rightarrow \quad \frac{\partial}{\partial t} n_{Y,5} \sim -\frac{1}{\pi}\alpha_Y \frac{\partial}{\partial t}\mathcal{H}_M$$

The exponential growth must saturate when the initial asymmetry is `used up'

helicity density: $\mathcal{H}_{M,\text{CPI}} \sim \frac{\pi}{6}\frac{1}{\alpha_Y}|\mu_{Y,5}|T^2_{\text{CPI}}$

coherence length: $\xi_{M,\text{CPI}} \approx 2\pi/(k_{\text{CPI}}/2)$

field strength: $B_{\text{CPI}} \approx \sqrt{\mathcal{H}_{M,\text{CPI}}/\xi_{M,\text{CPI}}}$

(We will validate these estimates using MHD simulations.)

using
$n_{Y,5} = \frac{1}{6}\mu_{Y,5}T^2$
$Y\tilde{Y} = -4\boldsymbol{E}_Y \cdot \boldsymbol{B}_Y = 4\frac{\partial}{\partial t}(\boldsymbol{A}_Y \cdot \boldsymbol{B}_Y) + \cdots$

Magnetogenesis

Typical scales

$$a_{\rm CPI} \simeq (1.1 \times 10^{-18} a_0) \left(\frac{g_*}{106.75}\right)^{1/6} \left(\frac{\eta_Y}{0.01 T^{-1}}\right)^{-1} \left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^{-2}$$

$$T_{\rm phys,CPI} \simeq (7.2 \times 10^4 \ {\rm GeV}) \left(\frac{g_*}{106.75}\right)^{-1/2} \left(\frac{\eta_Y}{0.01 T^{-1}}\right) \left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^{2}$$

$$d_{H,\rm CPI} \simeq (2.5 \times 10^{12} \ {\rm cm} \ a_0^{-1}) \left(\frac{g_*}{106.75}\right)^{1/3} \left(\frac{\eta_Y}{0.01 T^{-1}}\right)^{-1} \left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^{-2}$$

$$\xi_{M,\rm CPI} \simeq (5.0 \times 10^{5} \ {\rm cm} \ a_0^{-1}) \left(\frac{g_*}{106.75}\right)^{1/3} \left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^{-1}$$

$$B_{\rm CPI} \simeq (5.0 \times 10^{-11} \ {\rm G} \ a_0^{2}) \left(\frac{g_*}{106.75}\right)^{-2/3} \left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)$$

$$\Omega_{B,\rm CPI} \simeq (3.8 \times 10^{-10}) \left(\frac{g_*}{106.75}\right)^{-1} \left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^{2}$$

If $\mu_{Y5}/T \gtrsim 10^{-3}$ then the CPI develops before chiral charge erasure.

Inverse cascade

A maximally-helical field co-evolves with the plasma subject to the turbulent inverse-cascade:

$$B_0 = B_{\text{CPI}} \left(\frac{t_{\text{rec}}}{t_{\text{CPI}}} \right)^{-1/3} \quad \text{and} \quad \xi_{B,0} = \xi_{B,\text{CPI}} \left(\frac{t_{\text{rec}}}{t_{\text{CPI}}} \right)^{2/3} \quad \text{so} \quad \mathcal{H}_{B,0} = \mathcal{H}_{B,\text{CPI}}$$

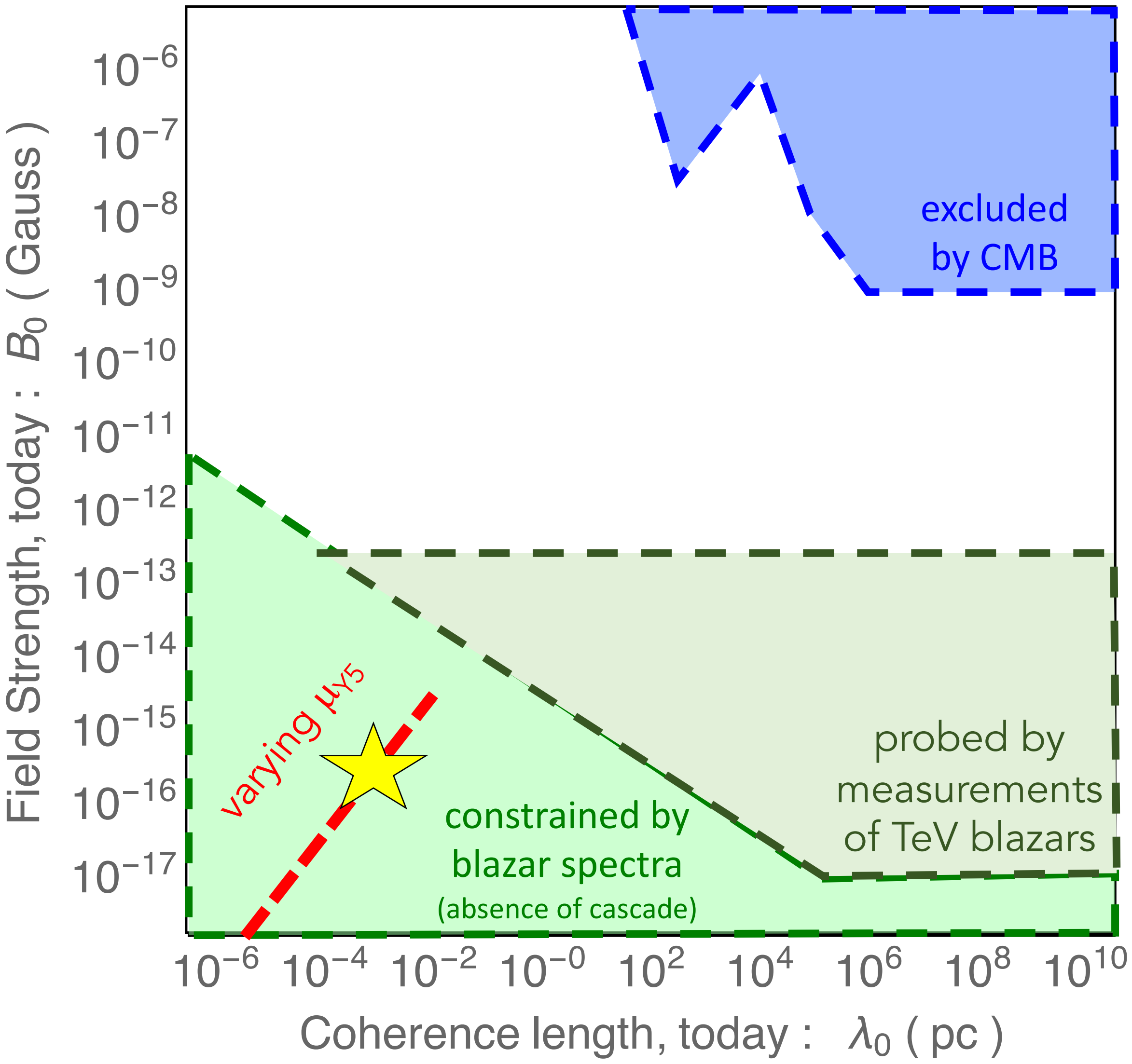
The field today is expected to be

$$\xi_{M,\text{phys},0} \simeq (9.5 \times 10^{-4}\ \text{pc}) \left(\frac{g_*}{106.75} \right)^{1/9} \left(\frac{\eta_Y}{0.01 T^{-1}} \right)^{2/3} \left(\frac{|\mu_{Y,5}|/T}{10^{-3}} \right)^{1/3}$$

$$B_{\text{phys},0} \simeq (6.6 \times 10^{-16}\ \text{G}) \left(\frac{g_*}{106.75} \right)^{-5/9} \left(\frac{\eta_Y}{0.01 T^{-1}} \right)^{-1/3} \left(\frac{|\mu_{Y,5}|/T}{10^{-3}} \right)^{1/3}$$

IGMF parameter space

adapted from: Durrer & Neronov (2013)



The relic of the primordial magnetic field is an intergalactic magnetic field today

$$\xi_{M,\mathrm{phys},0} \simeq (9.5\times 10^{-4}\ \mathrm{pc})\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^{1/3}$$

$$B_{\mathrm{phys},0} \simeq (6.6\times 10^{-16}\ \mathrm{G})\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^{1/3}$$

- viable seeds for galactic dynamo
- too weak to explain blazars

grav-wave generation
& stochastic GW background

Einstein's equation

for details: Roper Pol, Brandenburg, Kahniashvili, Kosowsky, & Mandal (2018)

The primordial magnetic field sources gravitational wave radiation

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\partial_t^2 h_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}$$

$$T_{ij} \sim B_i B_j \quad \text{(transverse / traceless part)}$$

$$\mathcal{E}_{\rm gw} = \langle \partial_t h_{ij} \partial_t h_{ij} \rangle / (32\pi G)$$

parametrically:

$$h \sim (GB^2)/\xi^2$$

$$\mathcal{E}_{\rm gw} \sim \xi^2 h^2 / G \sim G \xi^2 B^4$$

GW estimates

At the time of production

$$f_{\mathrm{gw,CPI}} \approx 2/\xi_{M,\mathrm{CPI}}$$
$$\mathcal{E}_{\mathrm{gw,CPI}} \sim (G/2\pi) a_{\mathrm{CPI}}^{-2} \xi_{M,\mathrm{CPI}}^2 B_{\mathrm{CPI}}^4$$

there's a 2 because the source is quadratic in the field

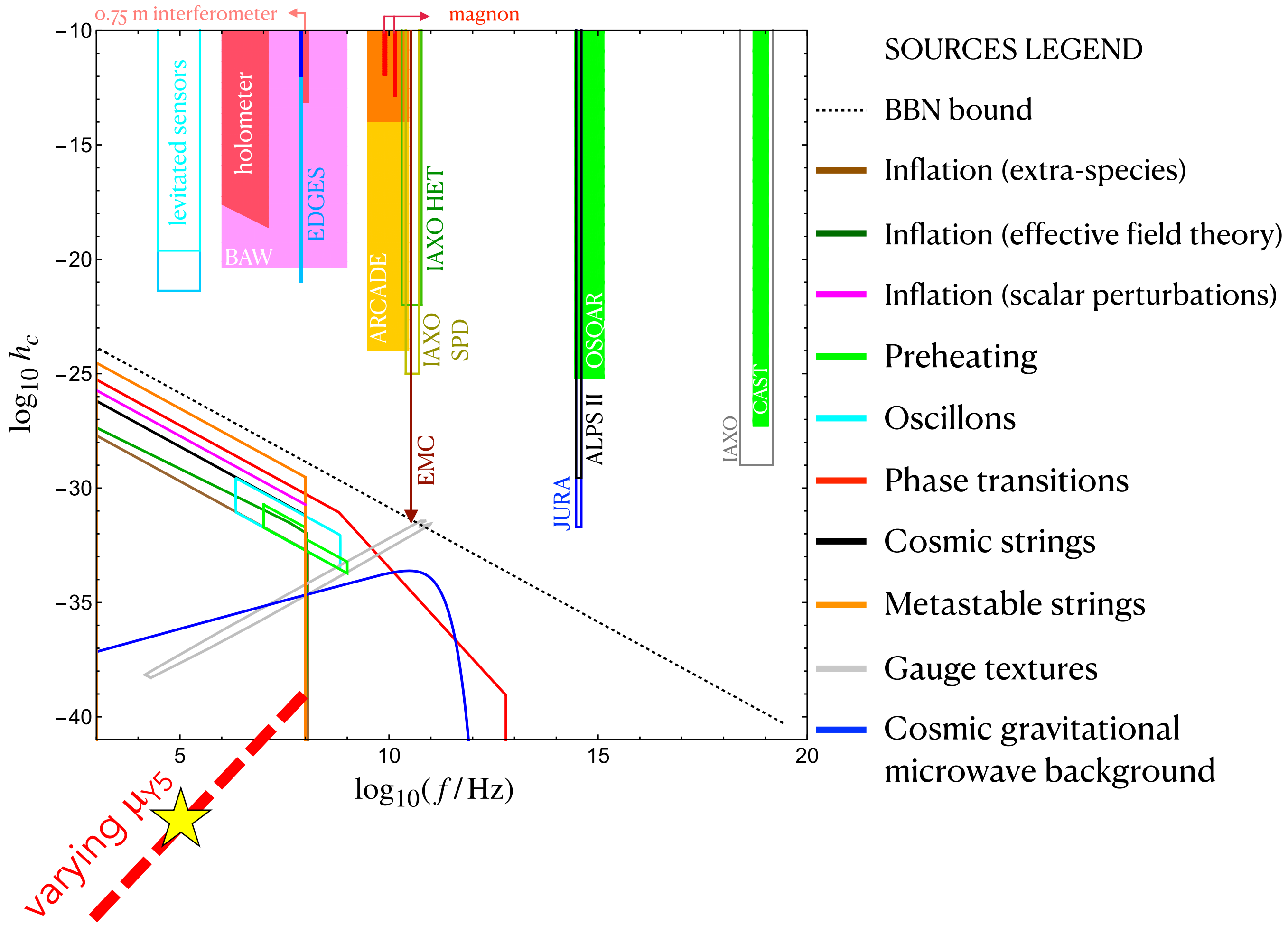
Frequency and cosmological energy fraction of stochastic GW today:

$$f_{\mathrm{gw,phys,0}} = \left(1 \times 10^5\ \mathrm{Hz}\right)\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)$$
$$(\Omega_{\mathrm{gw}} h^2)_0 = \left(7 \times 10^{-39}\right)\left(\frac{\eta_Y}{0.01 T^{-1}}\right)^2\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^6$$

The GW signal is high-frequency and very low amplitude (even for $\mu_{Y5}/T = 1$)

Searches for high-freq grav waves

review on high-freq grav wave: Aggarwal et al (2021)



$$f_{\mathrm{gw},0} = (1 \times 10^{5}\ \mathrm{Hz}) \left(\frac{|\mu_{Y,5}|/T}{10^{-3}} \right)$$

$$(\Omega_{\mathrm{gw}} h^2)_0 = (7 \times 10^{-39}) \left(\frac{|\mu_{Y,5}|/T}{10^{-3}} \right)^{6}$$

$$h_c = (1 \times 10^{-45}) \left(\frac{|\mu_{Y,5}|/T}{10^{-3}} \right)^{2}$$

using

$$\Omega_{\mathrm{gw}} = \frac{\pi f^2 h_c^2}{4 G \mathcal{E}_{\mathrm{cr}}}$$

numerical validation
with PENCIL CODE sims

MHD equations + CME

Fully nonlinear system of equations

$$\frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{u} \times \boldsymbol{B} + \eta(\mu_5 \boldsymbol{B} - \boldsymbol{J}), \quad (5)$$

$$\frac{\partial \mu_5}{\partial t} = -\boldsymbol{\nabla} \cdot (\mu_5 \boldsymbol{u}) - \lambda\eta(\mu_5 \boldsymbol{B} - \boldsymbol{J}) \cdot \boldsymbol{B} + D_5 \nabla^2 \mu_5, \quad (6)$$

$$\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} = \frac{2}{\rho}\boldsymbol{\nabla} \cdot (\rho\nu \mathsf{S}) - \frac{1}{4}\boldsymbol{\nabla} \ln \rho + \frac{\boldsymbol{u}}{3}(\boldsymbol{\nabla} \cdot \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \ln \rho) - \frac{\boldsymbol{u}}{\rho}\left[\boldsymbol{u} \cdot (\boldsymbol{J} \times \boldsymbol{B}) + \eta \boldsymbol{J}^2\right] + \frac{3}{4\rho}\boldsymbol{J} \times \boldsymbol{B}, \quad (7)$$

$$\frac{\partial \ln \rho}{\partial t} = -\frac{4}{3}(\boldsymbol{\nabla} \cdot \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \ln \rho) + \frac{1}{\rho}\left[\boldsymbol{u} \cdot (\boldsymbol{J} \times \boldsymbol{B}) + \eta \boldsymbol{J}^2\right], \quad (8)$$

$$\mathsf{S}_{ij} = (\partial_j u_i + \partial_i u_j)/2 - \delta_{ij} \boldsymbol{\nabla} \cdot \boldsymbol{u}/3$$

$$\frac{\partial^2}{\partial t^2}\tilde{h}_{+/\times}(\boldsymbol{k}, t) + k^2 \tilde{h}_{+/\times}(\boldsymbol{k}, t) = \frac{6\, H_*}{t\, \mathcal{E}_{\rm cr}} \tilde{T}_{+/\times}(\boldsymbol{k}, t),$$

magnetic diffusivity	η
kinematic viscosity	ν
chiral diffusion coeff.	D_5
rescaled chem. pot.	μ_5
chiral depletion param.	λ

PENCIL CODE
parallelized
6th order finite differences
3rd order time-stepping
1024^3 mesh points
initially: weak seed field, vanishing plasma velocity, homogenous chem. pot.

Parameters

cf, Brandenburg, He, Kahniashivili, Rheinhardt, & Schober (2021)

Dynamic range issues prevent studying the “expected” parameters.

Run	ηH_*	$(\mathcal{E}_{\rm cr}\lambda)^{1/2}/H_*$	μ_{50}/H_*	v_μ	v_λ	$\eta\mu_{50}^2/H_*$	k_1/H_*	$\mathcal{E}_{\rm M}^{\rm max}/\mathcal{E}_{\rm cr}$	$\mathcal{E}_{\rm GW}^{\rm sat}/\mathcal{E}_{\rm cr}$	q
B1	1×10^{-6}	2×10^{4}	10^4	1×10^{-2}	5×10^{-1}	1×10^{2}	1×10^{2}	1.6×10^{-2}	4.7×10^{-12}	0.027
B10	1×10^{-3}	2×10^{4}	10^4	1×10^{1}	5×10^{-1}	1×10^{5}	1×10^{2}	6.0×10^{-2}	6.0×10^{-9}	12
A1	1×10^{-6}	5×10^{4}	10^4	1×10^{-2}	2×10^{-1}	1×10^{2}	1×10^{2}	4.6×10^{-3}	8.9×10^{-14}	0.032
A12	5×10^{-3}	5×10^{4}	10^4	5×10^{1}	2×10^{-1}	5×10^{5}	5×10^{1}	9.2×10^{-3}	3.0×10^{-10}	18
X1	5×10^{-8}	10^{10}	10^6	5×10^{-2}	1×10^{-4}	5×10^{4}	5×10^{3}	2.4×10^{-9}	8.8×10^{-31}	0.39
X2	5×10^{-9}	10^{10}	10^6	5×10^{-3}	1×10^{-4}	5×10^{3}	5×10^{3}	2.4×10^{-9}	1.6×10^{-30}	0.53
X3	5×10^{-10}	10^{10}	10^6	5×10^{-4}	1×10^{-4}	5×10^{2}	5×10^{3}	2.4×10^{-9}	1.1×10^{-30}	0.44
X4	5×10^{-11}	10^{10}	10^6	5×10^{-5}	1×10^{-4}	5×10^{1}	5×10^{3}	2.3×10^{-9}	3.1×10^{-31}	0.12
Y1	5×10^{-8}	7×10^{11}	10^6	5×10^{-2}	1×10^{-6}	5×10^{4}	5×10^{3}	4.9×10^{-13}	3.6×10^{-38}	0.39
Y2	5×10^{-8}	7×10^{11}	10^6	5×10^{-2}	1×10^{-6}	5×10^{4}	2×10^{3}	4.4×10^{-13}	3.2×10^{-37}	1.3
Y3	5×10^{-8}	7×10^{11}	10^6	5×10^{-2}	1×10^{-6}	5×10^{4}	1×10^{3}	3.3×10^{-13}	6.9×10^{-37}	2.5
expected	$10^{-15}\eta_2$	6×10^{12}	$5\times10^{7}\mu_3$	$6\times10^{-8}\eta_2\mu_3$	$8\times10^{-6}\mu_3$	$3\eta_2\mu_3^2$	—	$6\times10^{-15}\mu_3^2$	$7\times10^{-39}\eta_2^2\mu_3^6$	—

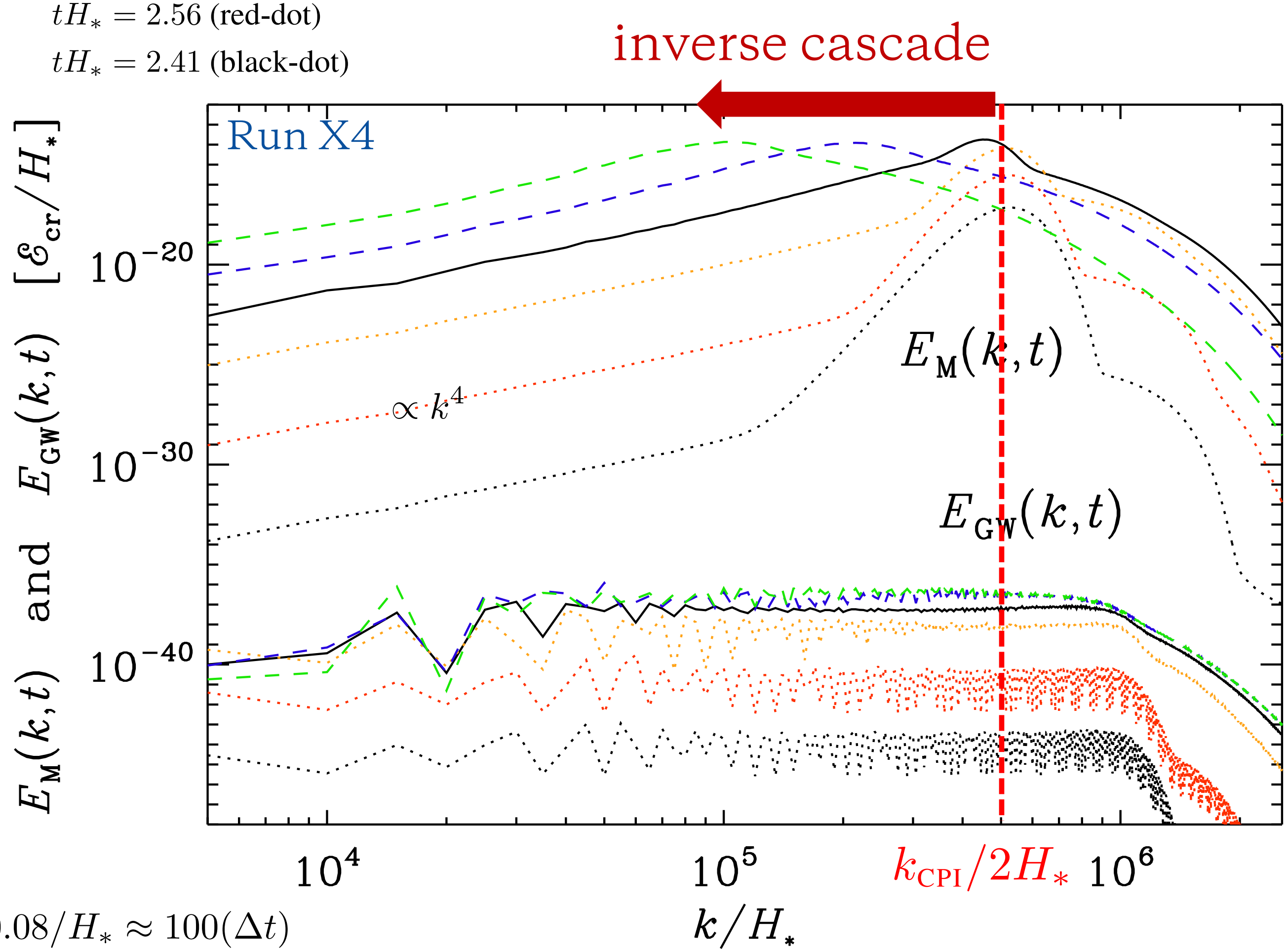
$$v_\mu = \mu_{50}\eta$$
$$v_\lambda = \mu_{50}/(\mathcal{E}_{\rm cr}\lambda)^{1/2}$$

$$\mu_{50} = 2\alpha_Y\tilde{\mu}_{Y,5}/\pi$$
$$\lambda = 192\alpha_Y^2/T^2$$
$$D_5 = \nu = \eta = 1/\sigma$$

$$\eta_2 = \eta_Y/(0.01T^{-1})$$
$$\mu_3 = \tilde{\mu}_{Y,5}/(10^{-3}T)$$

Spectra for PMF and GWs

$tH_* = 5.37$ (green-dashed)
$tH_* = 3.66$ (blue-dashed)
$tH_* = 2.98$ (solid)
$tH_* = 2.71$ (orange-dot)
$tH_* = 2.56$ (red-dot)
$tH_* = 2.41$ (black-dot)



$t_{CPI} = 0.08/H_* \approx 100(\Delta t)$

NB : $\mathcal{E}_M(\boldsymbol{x},t) = \frac{1}{2}\langle|\boldsymbol{B}(\boldsymbol{x},t)|^2\rangle = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} P_B(\boldsymbol{k}) = \int \frac{dk}{k} \underbrace{\frac{k^3}{2\pi^2} P_B(k)}_{= d\mathcal{E}_M/d\ln k} = \int dk\, E_M(k)$

- mag energy grows at $k_{CPI}/2$
- GW energy grows while mag energy grows
- mag energy reaches maximum and GW energy spectrum saturates (solid curves)
- subsequently mag energy evolves with the inverse cascade scaling (expected for helical):

$$B \propto t^{-1/3} \quad \text{so} \quad \mathcal{E}_M \propto t^{-2/3}$$

$$\xi_M \propto t^{2/3} \quad \text{so} \quad k_M \propto t^{-2/3}$$

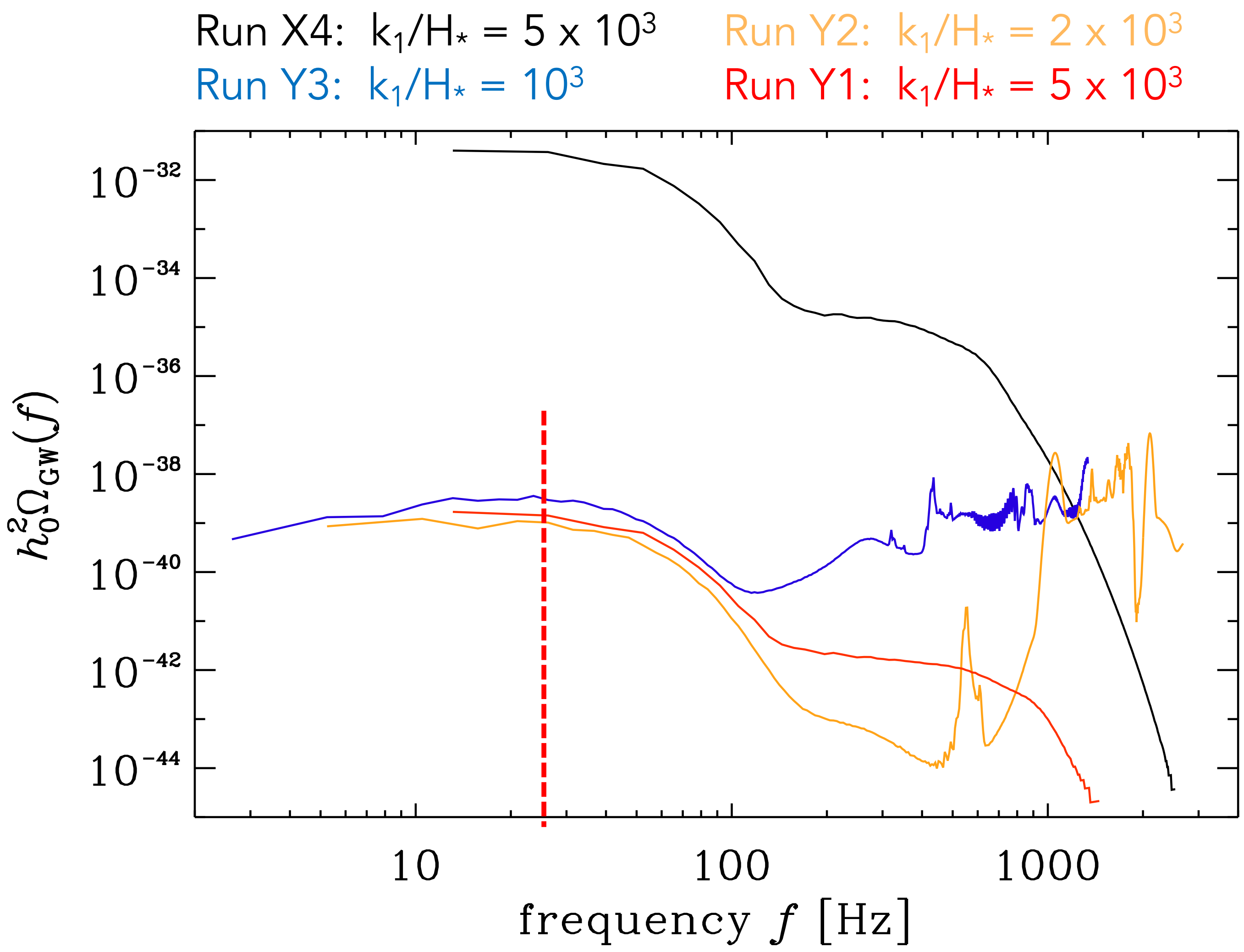
$$E_M \sim \mathcal{E}_M / k_M \propto t^0$$

- resultant GW energy spectrum is blue-tilted

$$\frac{d\mathcal{E}_{gw}}{d\ln k} = kE_{GW} \propto k^1 \quad (k \lesssim k_{CPI})$$

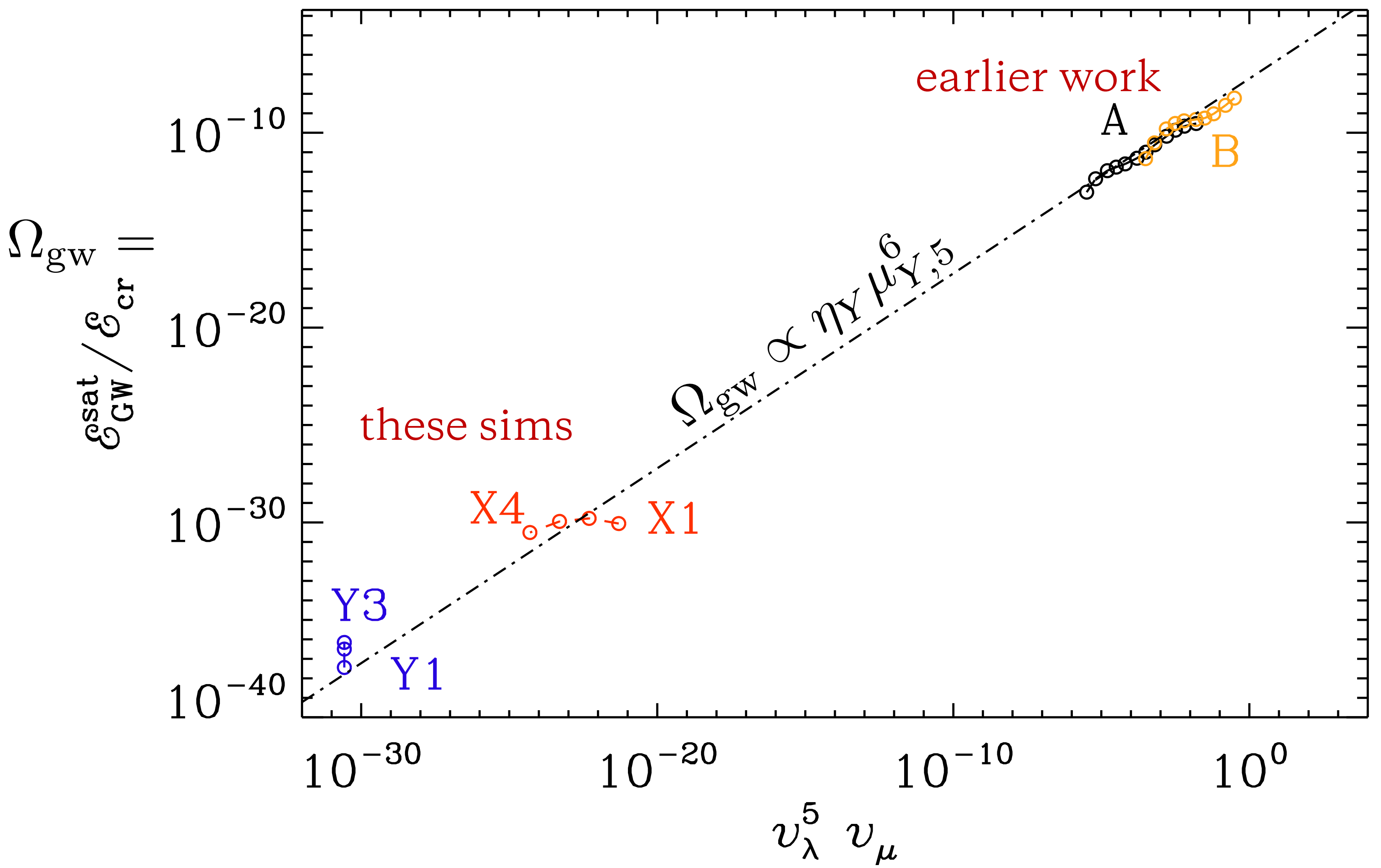
  - (scaling should extend down to k_{IR}~H)
  - (most energy at k_{UV}~k_{CPI})

Grav wave frequency spectrum today



- a high-frequency & low amplitude
- UV end of the spectrum suffers from numerical instability (that depends on the k_1, highest wavenumber on the lattice)

Comparision w/ previous studies



Earlier work studied GW from CME at the EW epoch, assuming no chiral asymmetry washout (see Run Series A & B)

We explore parameters that are closer to the expectation for early universe cosmology at T > 80 TeV (see Run Series X and Y)

Main difference is the choice of magnetic diffusivity: η_Y. (see the table on prev. slide)

For those parameters, the GW signal is found to be weaker.

Scaling with $v_\lambda^5 \, v_\mu$ from earlier work, confirmed over wider range.

cf., Kohei & Kyohei's talks

B-number overproduction
& an upper bound on μ_{Y5}

Sourcing baryon number

Fujita & Kamada (2016), Kamada & AL (2016a,b),
Jimenez, Kamada, Schmitz,Xu (2017)
see also: Giovannini & Shaposhnikov
(also gravitational anomaly: cf, Evangelos's talk)

Varying hypermagnetic helicity sources B and L-number

$$\partial_\mu j^\mu_{\mathsf{B}} = \partial_\mu j^\mu_{\mathsf{L}} = 3\frac{g^2}{16\pi^2} W^a \tilde{W}^a - 3\frac{g'^2}{16\pi^2} Y\tilde{Y}$$

$$\dot{n}_{\mathsf{B}} = \dot{n}_{\mathsf{L}} = -3\frac{g'^2}{4\pi^2}\dot{\mathcal{H}}_Y + \cdots$$

At the EW crossover, there is a change in helicity when the B_Y field converts to B_{em}

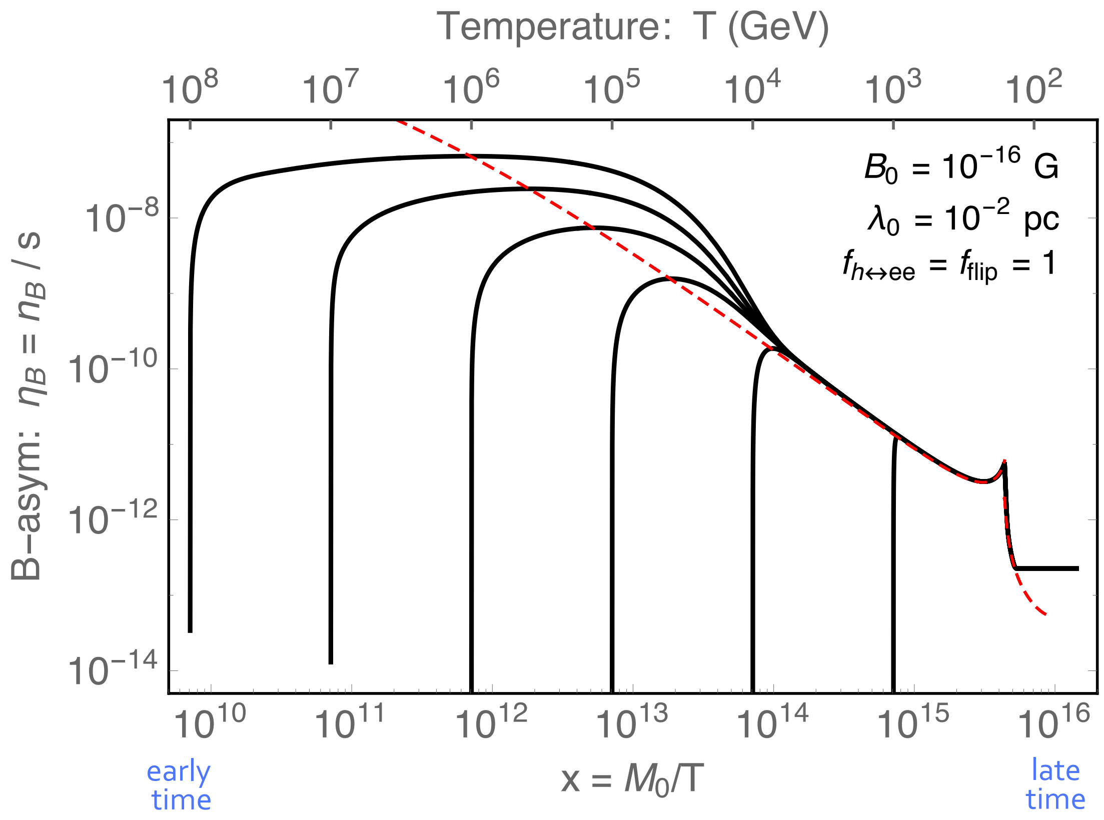
- ➔ this could explain the matter/antimatter asymmetry!
- ➔ but it would be a problem if too many baryons are created

(B+L) washout is avoided

Kamada & AL (2016)

basic idea: $\dot{n}_\mathrm{B} = -\Gamma_\mathrm{sphaleron}\, n_\mathrm{B} + S_\mathrm{helicity}$ ⇨ $n_\mathrm{B} \sim S_\mathrm{helicity} \,/\, \Gamma_\mathrm{sphaleron}$

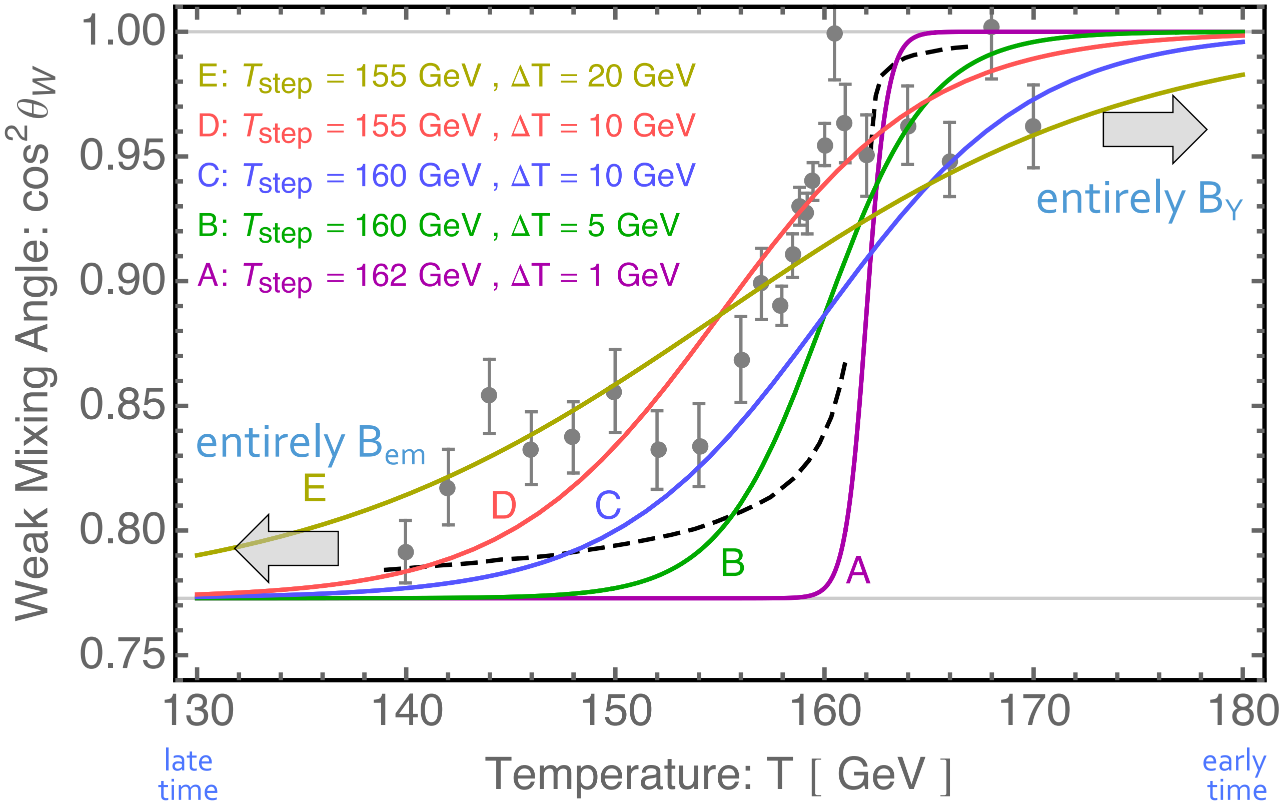
time evolution



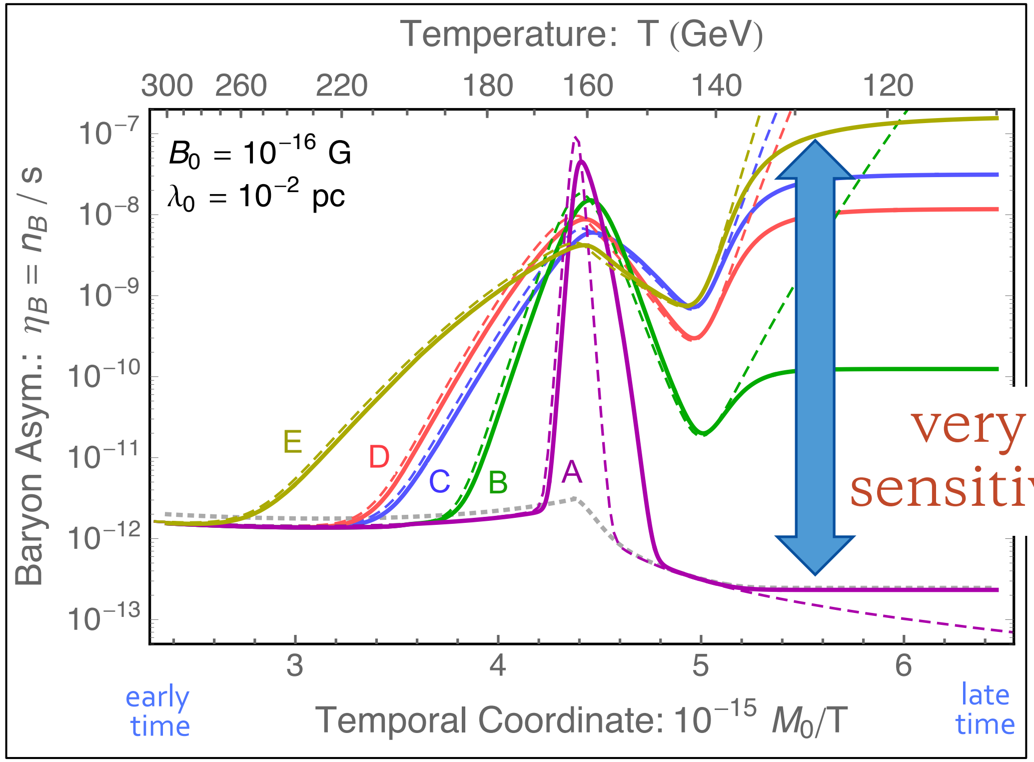
Sensitivity to the EW crossover

Kamada & AL (2016)

Z-γ mixing is proxy for B_Y to B_{em}



time evolution



We should work to develop a better understanding of how hypermagnetic fields are converted into electromagnetic fields at the EW epoch (even if it is a smooth crossover)

Avoiding baryon overproduction

[Domcke, Kamada, Mukaida, Schmitz, Yamada (2022)]
cf., Kyohei's talk

If μ_{Y5} is too large, the helical B_Y-field becomes too strong, and B-number is over-produced.

The upper limit is subject to uncertainties associated with modeling B_Y -> B_{EM} at the EW crossover.

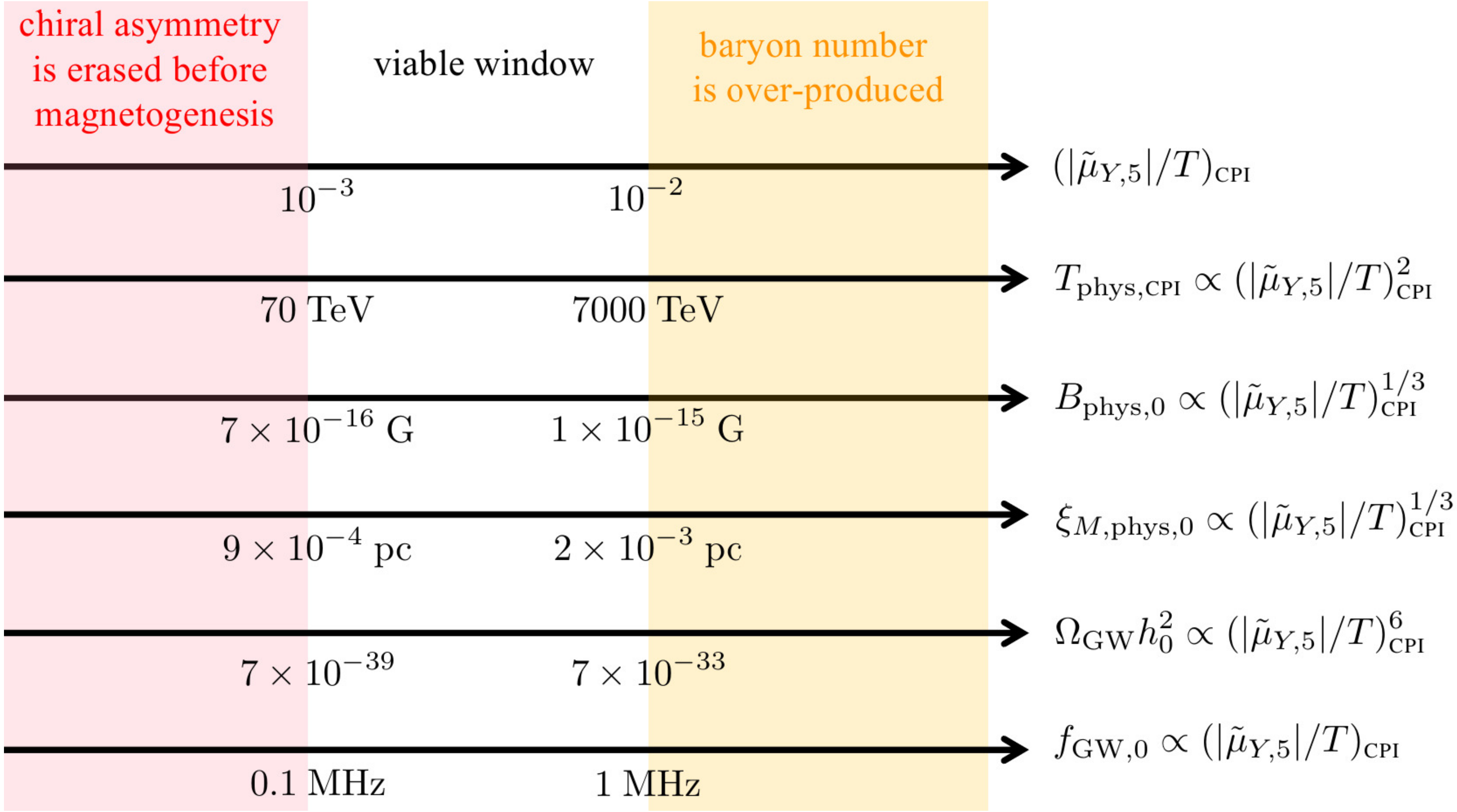
We take $\mu_{Y,5}/T \lesssim 10^{-2}$ as a guide.

recall

$$B_{\mathrm{phys},0} \simeq (6.6 \times 10^{-16}\ \mathrm{G}) \left(\frac{|\mu_{Y,5}|/T}{10^{-3}} \right)^{1/3}$$

summary graphic
& remarks

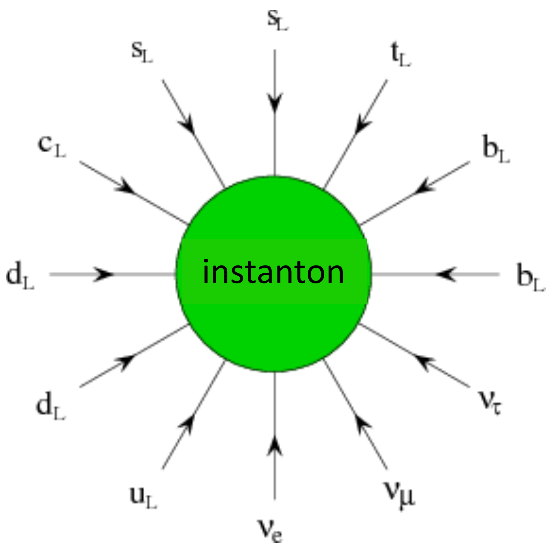
A narrow viable window for early universe CPI



the end of it all
(exotic nucleon decay)

What are the “observable” consequences of Standard Model anomalous B- and L-number violation? Does it lead to proton decay?

$$\partial_\mu j^\mu_{\mathsf{B}} = \partial_\mu j^\mu_{\mathsf{L}} = 3\frac{g^2}{16\pi^2} W^a \tilde{W}^a - 3\frac{g'^2}{16\pi^2} Y\tilde{Y}$$



VOLUME 37, NUMBER 1 PHYSICAL REVIEW LETTERS 5 JULY 1976

Symmetry Breaking through Bell-Jackiw Anomalies*

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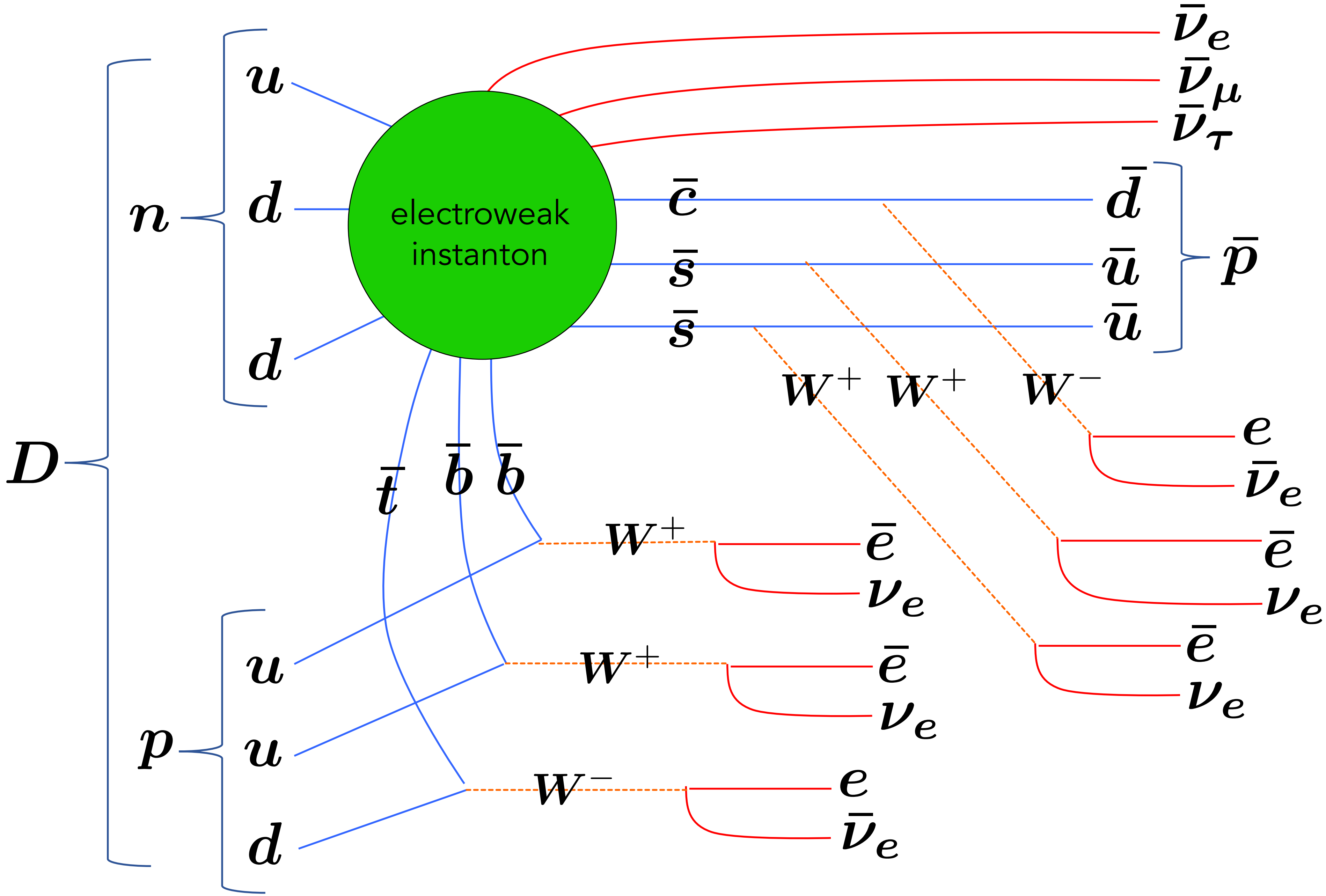
(Received 22 March 1976)

In models of fermions coupled to gauge fields certain current-conservation laws are violated by Bell-Jackiw anomalies. In perturbation theory the total charge corresponding to such currents seems to be still conserved, but here it is shown that nonperturbative effects can give rise to interactions that violate the charge conservation. One consequence is baryon and lepton number nonconservation in $V-A$ gauge theories with charm. Another is the nonvanishing mass squared of the η.

Thus, because of the Cabibbo rotation, a proton and a neutron (two baryons equal six quarks) may annihilate to form two antileptons, one of electron and one of muon type.

Since ΔB = +-3, proton decay is kinematically blocked.
But the deuteron (D=pn) will decay.

Standard Model deuteron decay



An immensely rough rate estimate

$$D \to \bar{\nu}_e + \bar{\nu}_\mu + \bar{\nu}_\tau + \bar{p} + 4\bar{e} + 2e + 4\nu_e + 2\bar{\nu}_e$$

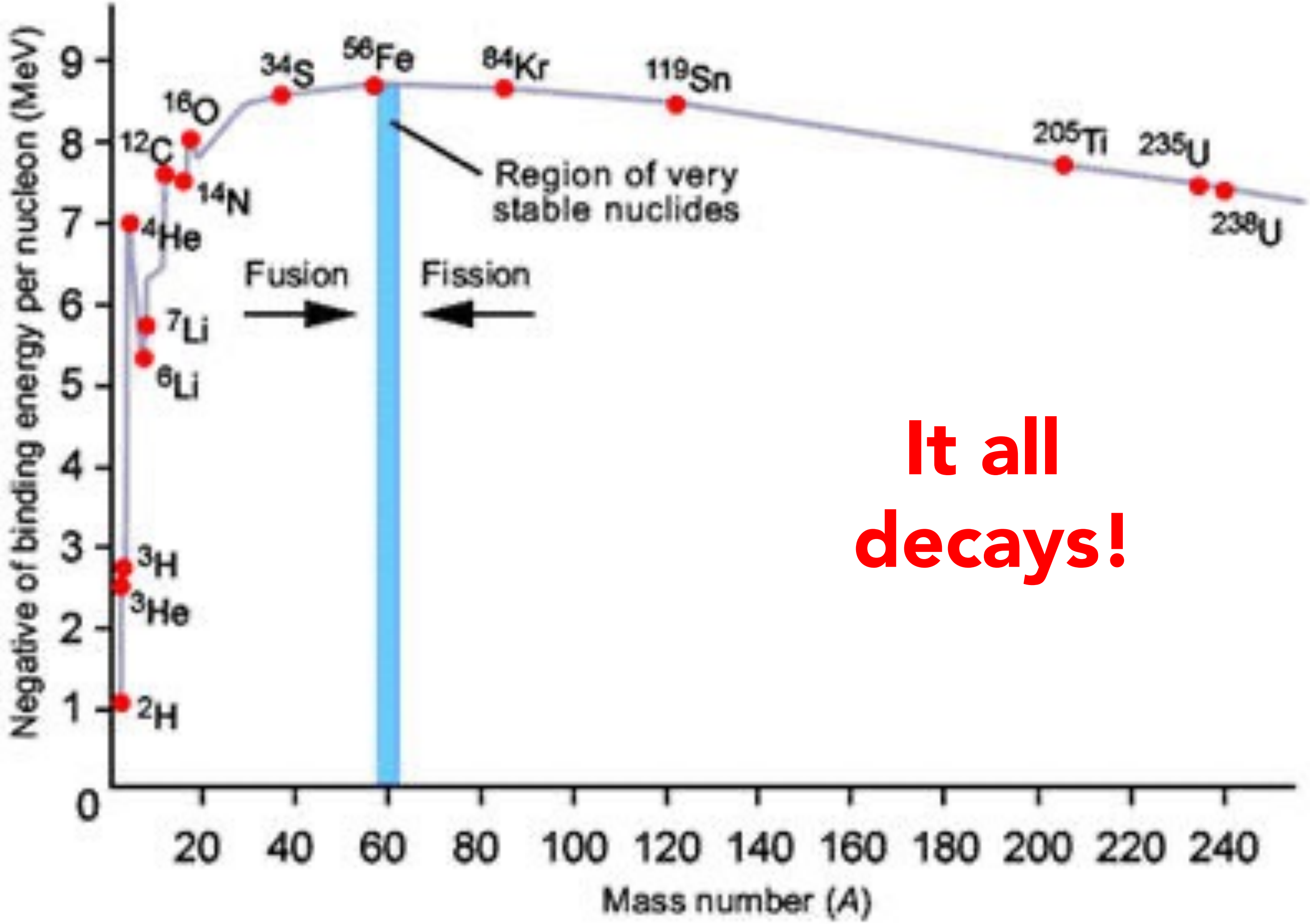
$$\left(\Delta Q = 0\,, \quad \Delta B = -3\,, \quad \Delta L = -3 \right)$$

A very rough rate estimate ...

$$\Gamma \sim G_F^{12}\, m_D^{25}\, V_{td}^2\, V_{ub}^4\, V_{cd}^2\, V_{us}^4\, e^{-16\pi^2/g^2}$$

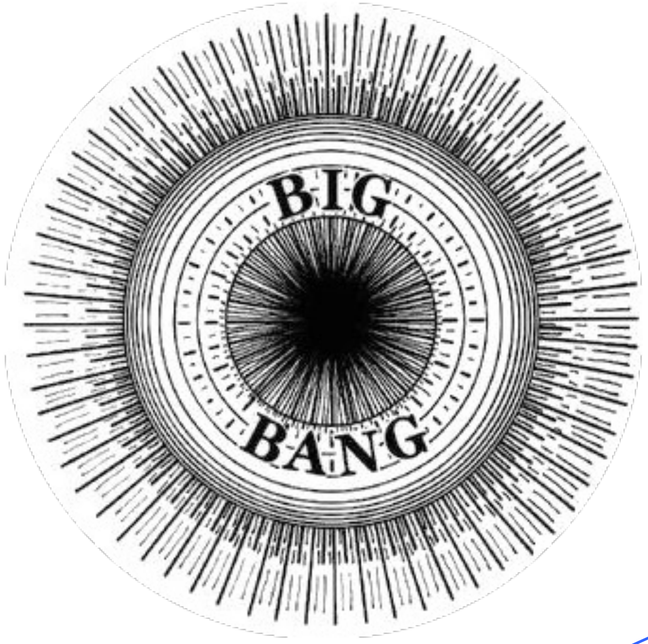
$$\tau_D \sim 10^{184}\,\mathrm{yr}$$

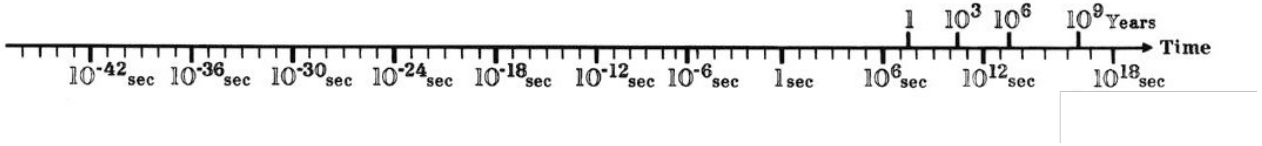
Heavier nuclei are not safe either!

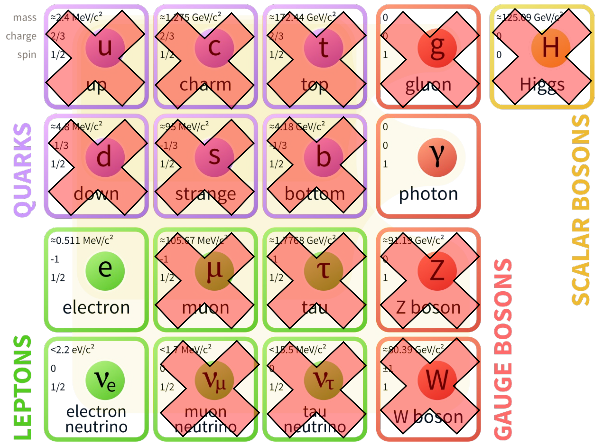


It all decays!

What's at the end of the cosmic timeline?







1 yr 10^{20} 10^{40} 10^{60} 10^{80} 10^{100} 10^{120} 10^{140} 10^{160} yr

p decay (quantum gravity)

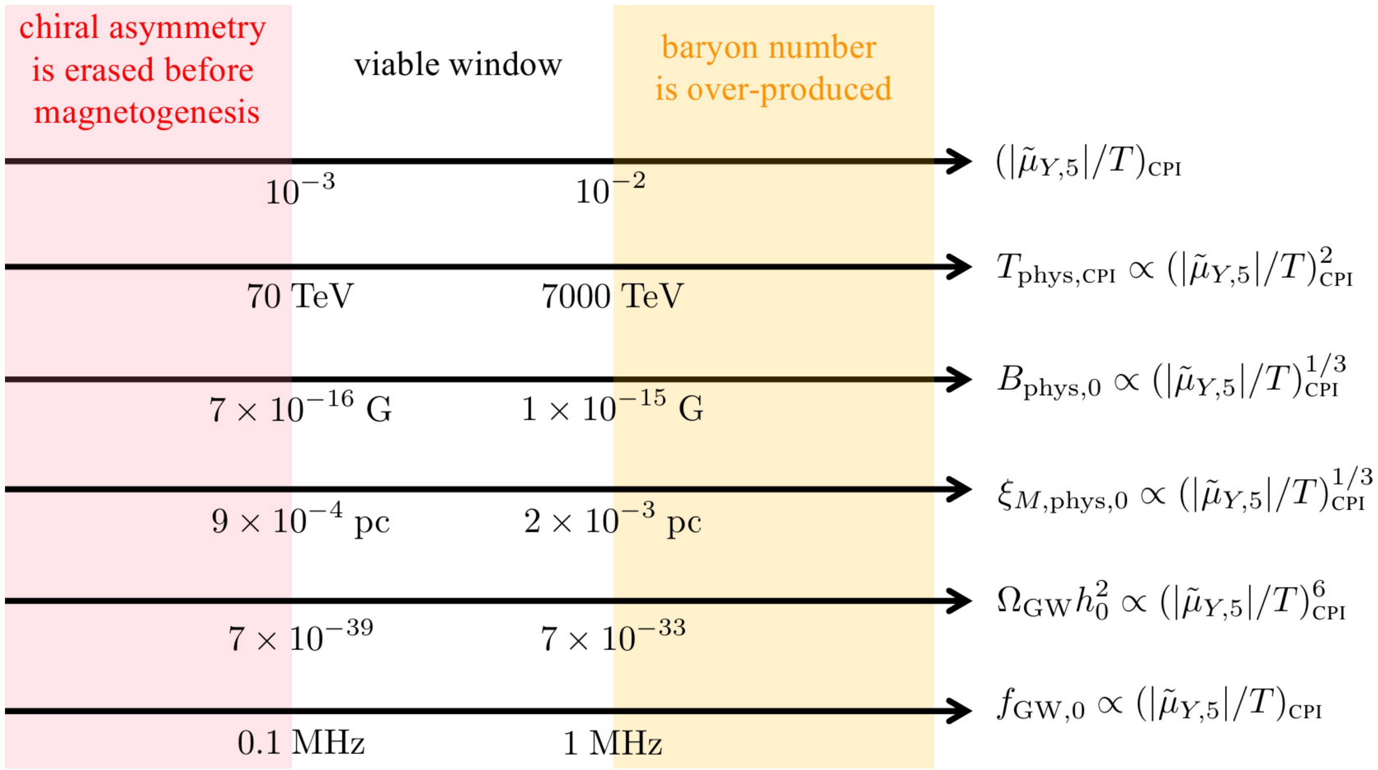
ν decay (weak intn's)

D & nuclei decay (instantons)

summary
& conclusion

Summary

CPI ➔ PMF ➔ GWs



- chiral plasma instability develops at T_{CPI} > 80 TeV
- development of CPI leads to helical PMF and chiral GWs
- PMF could provide seeds for dynamo – cannot account for blazars
- GWs are high frequency and too weak to detect
- interesting to consider: CPI after 80 TeV with a source

backup slides

Stochastic B-field power spectra & energy

$$\langle B_i(\boldsymbol{k},t) B_j^*(\boldsymbol{k}',t)\rangle = (2\pi)^3\,\delta(\boldsymbol{k}-\boldsymbol{k}')\left[\left(\delta_{ij}-\hat{k}_i\hat{k}_j\right)P_B(k) - \mathrm{i}\epsilon_{ijm}\hat{k}_m P_{aB}(k)\right]$$

$$\langle B_i(\boldsymbol{x},t) B_j(\boldsymbol{y},t)\rangle = \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\frac{\mathrm{d}^3\boldsymbol{k}'}{(2\pi)^3}\langle B_i(\boldsymbol{k},t) B_j^*(\boldsymbol{k}',t)\rangle\,\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}-\mathrm{i}\boldsymbol{k}'\cdot\boldsymbol{y}}$$

$$= \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\left[\left(\delta_{ij}-\hat{k}_i\hat{k}_j\right)P_B(k) - \mathrm{i}\epsilon_{ijm}\hat{k}_m P_{aB}(k)\right]\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})}$$

$$\mathcal{E}_M(\boldsymbol{x},t) = \frac{1}{2}\sum_i \langle B_i(\boldsymbol{x},t) B_i(\boldsymbol{x},t)\rangle$$

$$= \frac{1}{2}\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\left[\left(3-1\right)P_B(k)\right]$$

$$= \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} P_B(k)$$

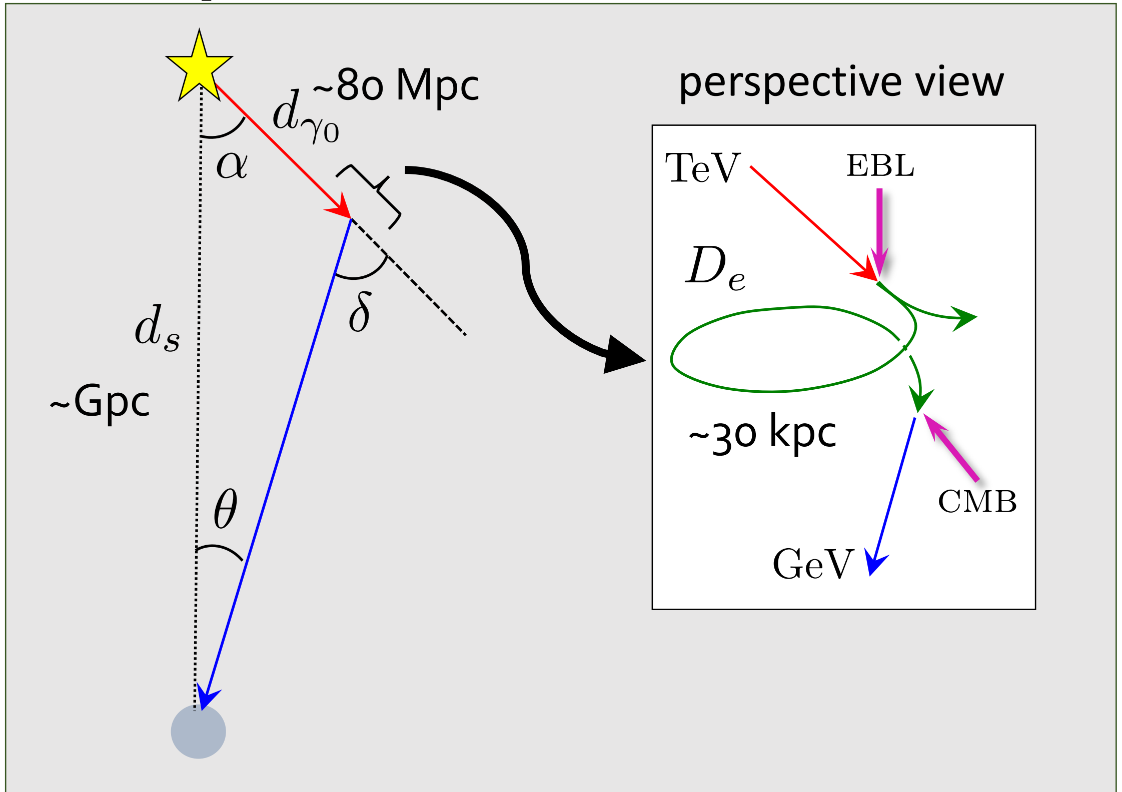
$$= \int_0^\infty \frac{\mathrm{d}k}{k}\,\underbrace{\frac{k^3}{2\pi^2}P_B(k)}_{=\,kE_M(k)}$$

TeV blazars
as a probe of the IGMF

[Aharonian, Coppi, & Voelk (1994); Neronov & Semikoz (2006); AL & Vachaspati (2016)]

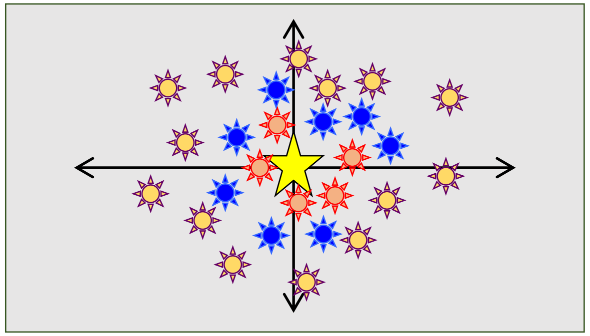
Gamma rays from TeV blazars develop an electromagnetic cascade by scattering on starlight (EBL) and cosmic microwave background (CMB) photons.

The presence of an IGMF deflects the cascade.

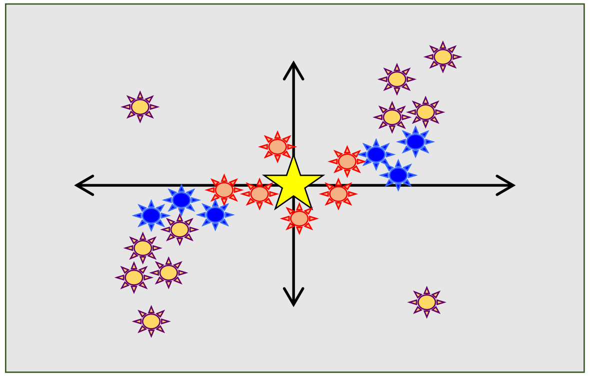


$$\Theta_{\rm ext} \simeq (0.68^\circ) \left(\frac{B_0}{10^{-15}\,{\rm G}}\right) \left(\frac{E_\gamma}{10\,{\rm GeV}}\right)^{-3/2} \left(\frac{d_s}{1\,{\rm Gpc}}\right)^{-1}$$

the blazar acquires a halo



the blazar halo becomes “twisted” by a *helical* IGMF



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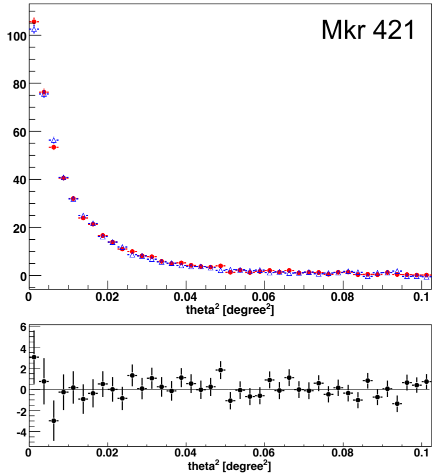
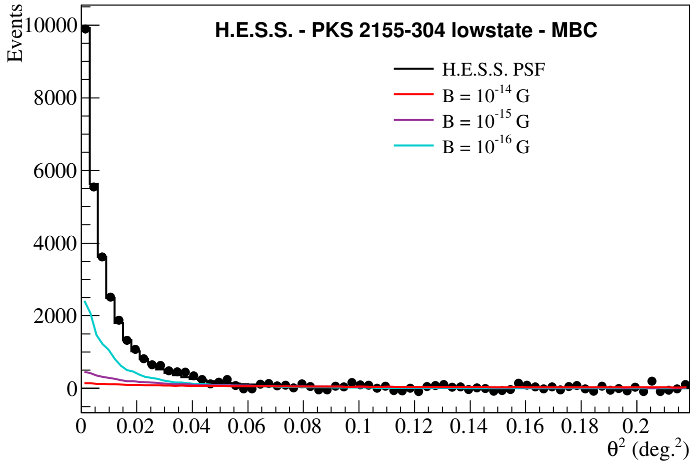
Ongoing experimental efforts …

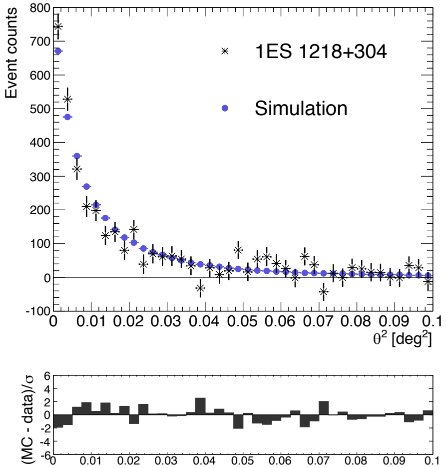
MAGIC (2010)



HESS + Fermi-LAT (2014)



VERITAS (2017)



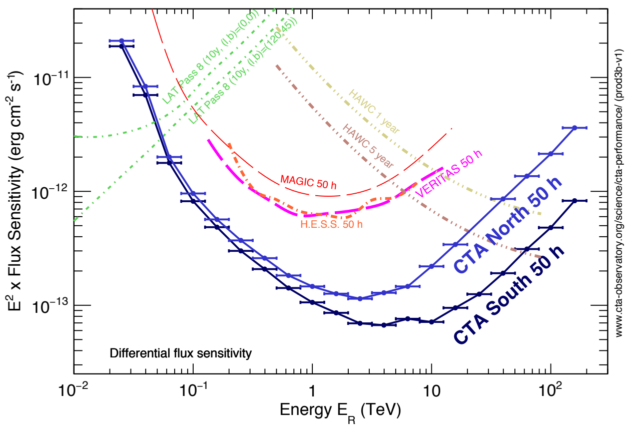
A halo is not observed ➔ some of the IGMF parameter space is excluded:

$$B_0 \sim (0.3 - 70) \times 10^{-15}\,\mathrm{G} \quad \mathrm{for} \quad \lambda_0 = 1\,\mathrm{Mpc}$$

[Meyer, Conrad, & Dickinson (2016); CTA Science Book (2017)]

On the horizon…

The Cherenkov Telescope Array (CTA) will dramatically improve the flux sensitivity & angular resolution (PSF).



Implications for IGMF contraints:

Benchmark models (3, green). Corresponding exclusions (blue).